Effect of particle breakage on the behavior of simulated angular particle assemblies

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Received 14 January 2007; accepted 26 June 2007

Abstract

Many attempts have been made to find various relationships for different parameters and some kinds of constitutive models for studying the behavior of particulate media. All these models are based on concepts of continuous media. Using a numerical method such as discrete element method, one can figure out what is happening through a discontinuous media where soil particles play the main role in introducing the shear strength and deformation characteristics. The behavior of the media with breakable particles is studied in this paper and compared with that of the assembly with non-breakable particles. In this paper, the hyperbolic elastic model is investigated for the assembly of polygon shaped particles in two different test series. In addition, evolution of different macro parameters of the assembly such as volume strain, angle of friction, angle of dilatancy and elastic modulus is studied during the simulation tests both for non-breakable and breakable soil particles. At the end, a parametric study is performed on the effect of strength of particle breakage on the assembly behavior.

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Keywords: Discrete element modeling; Particle breakage; Hyperbolic elastic model; Confining pressure

1. Introduction

The crushability of soil particles has been studied experimentally by a number of researchers, and the influence of particle breakage on internal friction angle and deformability of granular materials has been investigated with tests such as triaxial and unconfined compression tests (Bertacchi & Bellotti, 1970; Fumagali, Moscini, & Rossi, 1970; Marachi, Chan, & Seed, 1972; Marsal, 1967; Varadarajan, Sharma, Venkatachalam, & Gupta, 2003; Venkatachalam, 1993).

Because of grain crushability, triaxial tests of sands show strong dependence of strength and dilatancy behavior on both relative density and stress level (Bolton, 1986). An empirical characterization of peak angle of internal friction of sands was reasonably successful, although inherent anisotropy due to bedding was also seen to be important.

Breakage of particles is very important as it influences the stability and the deformation of earthwork structure where there is high pressure in the lower parts of the structure. This happens especially in the underlying layers which bear the weight of the upper layers. The soil grains in the underlying layers are subjected to significant stress magnitudes, and the induced high stresses may cause the particles to break. Particle breakage results in change in the grain size (gradation) curve, leading to the alteration of the mechanical behavior of the granular material. In general, shear resistance and behavior of granular materials depend on different factors such as mineralogical composition, particle grading, size and shape, as well as their fragmentation and stress conditions. Here, we only emphasize particle crushing.

In recent years, along with progress in computer technology, different methods have been developed to model breakage of brittle bodies with the help of discrete element method (DEM). Discrete element simulation of perfectly elastic and infinitely strong grains provides insight into the deformation of granular media (Thornton, 2000). Among these methods, are those based on simultaneous utilization of molecular dynamics (MD) (Kun & Herrmann, 1996) and the 3D approach used by Robertson and Bolton (2001) and McDowell and Harireche (2002).

Although the aforementioned researches well studied the fragmentation of particulate media, they are highly time-consuming due to the large number of elements in the
simulations. The present study tries to model particle breakage by using less number of particles and employing a simple mechanism for breakage (Mousavi Nik, 2000), focused on the influence of particle breakage on the behavior of granular media. The classical nonlinear elastic hyperbolic model is chosen to investigate particulate media with and without particle breakage by using DEM simulation of biaxial tests.

2. Brief review of modeling

The program POLY (Mirghasemi, Rothenburg, & Matyas, 1997), a modified version of DISC (Bathurst, 1985) for simulating two-dimensional polygon-shaped particles, has been developed to model assemblies of irregularly shaped particles, capable of dealing with breakage (Seyedi Hosseininia & Mirghasemi, 2006). In the present research, a modified program called M-POLY was used to perform simulation of biaxial test on assemblies of 500 particles within 1500 sub-particles using a personal computer.

In this method it is assumed that each soil particle can break along pre-defined straight lines with certain direction and position. The lines are determined in a way that two kinds of commonly observed behavior can be simulated. These two kinds of behavior are cracking of particle vertices and cracking across a particle that divides particle into pieces. For example, according to Fig. 1(a), it is assumed that the particle P can only break along the lines d1, d2 and d3; therefore shape of the particles obtained from breakage of the primitive particle is specified from the beginning. Thus, in this method, each uncracked particle like P consists of smaller bonded particles like P1, P2, and Pn. Particle P is called the base particle and the particles P1 through Pn are called sub-particles. The sub-particles are considered rigid bodies, neither breakable nor deformable. Base particles are not deformable but breakable. Both base and sub-particles are arbitrary convex polygon shaped. In order to ensure the rigidity and continuity of bonded particles in a base particle, it is assumed that two adjacent sub-particles are connected with each other by a fixed connection at the middle of their common edge (points m1 and m2 in Fig. 1(b)). This fixed connection plays the role of a bond between two bonded sub-particles, capable of withstanding shear, tension, compression and finally bending forces. If at a specific moment during simulation, the stress applied to the connection exceeds its final bearing capacity, the connection will break and the two bonded particles thus separate from each other, that is, breakage takes place. The detailed method for simulating the bond between these sub-particles is presented in references (Seyedi Hosseininia & Mirghasemi, 2006).

The inter-particle friction coefficient is set to 0.5 for all tests ($\phi_{\mu} = 26.6^{\circ}$) and the particles are assumed to be cohesionless at the contact as well as weightless. The other parameters used in the simulation tests are referred to Seyedi Hosseininia & Mirghasemi (2006).

For an assembly containing particles like that shown in Fig. 1, on particles located at the assembly boundary of the medium, forces and displacements are imposed. After loading in each step, the inside particles experience contact forces from the adjacent particles and thus try to move and then reach a balanced state. For an assembly with particles in equilibrium in terms of the internal and external forces on each particle, it was shown that the stress tensor in such an assembly can be calculated in a 2D space from the imposed forces on the internal particles (Bathurst, 1985; Cundall, 1978; Rothenburg, 1980) as follows:

$$\sigma_{ij} = \frac{1}{A} \sum_{s \in A} f_{is}^{j} l_{s}^{j}, \quad i, \ j = 1, 2. \quad (1)$$
Regarding the calculation of strain in the assembly, the relative displacement of each particle on the boundary can give us the strain of the assembly in the corresponding simulation step. The strain values mentioned here correspond to the average strains of the boundary particles in two directions, 1–1 and 2–2, as shown in Fig. 2. The contact forces are assessed from the contact law presented by Mirghasemi et al. (1997).

3. Test simulations

Simulation of two series of biaxial compression tests is fulfilled under five levels of confining pressure of 0.5, 1.0, 2.0, 4.0 and 8.0 MPa to investigate the particle breakage in a granular medium. In test series A particles are rigid and unable to fragment while in test series B, the rigid particles are breakable. The area that the particles are held in is a circle.

Apart from generating the initial assembly of particles (Fig. 2(a)), each test simulation includes three stages. Since particles are away from each other in the initially generated assembly, the assembly is first compacted to decrease the existing large void ratio. Then, the assembly is subjected to a pre-defined confining pressure to hold the particle assembly together (Fig. 2(b)). Confining pressure can be changed in each simulation set to study the effect of confining pressure on the assembly behavior. Finally the assembly is subjected to a shear strain in such a way that the stress in direction 1–1 remains constant and the strain in direction 2–2 varies with a constant strain rate of 0.005 (Fig. 2(c)). After each stage of simulation, the assembly needs to remain stable, i.e. particles do not move while there is no unbalanced force acting on them. Three stages of simulation for a breakable test are shown in Fig. 2. The test simulations are fulfilled up to an axial strain level between 16% and 18%.

4. Test results

The results of biaxial test simulations at a confining pressure level of 2.0 MPa ($\sigma_{c.p.} = 2.0$ MPa) are presented in terms of stress obliquity ($\sigma_2/\sigma_1$) and volumetric strain versus axial strain (Fig. 3), respectively. As shown in Fig. 3(a), the shear strength (stress obliquity) in test series A increases rapidly at the initial stage as strain increases, then it reaches a peak value gradually and finally decreases and becomes constant later. In test B, the trend is different where the stress ratio increases gradually till a constant value. It seems that particle breakage has a decreasing effect on shearing resistance of the assembly.

In both tests, the shear strength is mobilizing as the axial strain increases and their difference in behavior is due to the occurrence of interlocking. That is why we can observe some peak values in test A but no more in test B. Particles in test A are nonbreakable, and therefore may interlock with each other, causing a peak strength in the behavior followed by a reduction when this interlocking collapses and particles are likely to slide on each other with their inter-particle resistance angle ($\phi$).

Hence, the shear resistance of soil mass is made up of two components: one is controlled by $\phi$ and the other is related to the degree of interlocking. The greater the degree of interlocking, the higher the overall shear resistance. Moreover, as shear motion continues, the interlocking decreases and consequently, the shear force necessary to sustain the motion must also decrease (in large strains).
In contrast to test A, the peak shear strength is not seen in test B since particles cannot bear the implied stresses and neither can interlocking happen, but the broken particles tend to show the largest power of strength at the large strains where the deformation is so large that the structure may have already been damaged.

In general, angular particles exhibit dilative behavior. The fact that the volume of a dense soil sample increases while compressed in one direction, was first observed by Osbourne Reynolds in 1885. Marsal (1967) by performing triaxial compression tests on coarse granular materials found out that the most important factor affecting both shear strength and compressibility is the phenomenon originating from displacements between particles as well as the particle breakage. Varadarajan et al. (2003) investigated the behavior of two dam site rock materials in triaxial compression tests consisting of rounded and angular particles, respectively. Although they found the particle breakage in both materials, the shear strength of the assembly with angular particles was higher with the dilation in volume after initial compression. However, the rounded material exhibited continuous volume compression accompanied by lower shear strength. The same result is obtained in the simulation tests.

Fig. 3(b) shows the volumetric strain of both test series during biaxial test. By comparing values of volumetric strain in both groups, it seems that the assembly with nonbreakable particles is more dilative than that with breakable particles. In other words, the more the assembly dilates, the larger its shear resistance is. In test B, particles cannot bear the forces imposed on them and breakage happens, therefore smaller particles fill the voids letting other particles move freely. This trend can justify the reduction of stress ratio in test B.

Fig. 4 illustrates the variation of bond breakage degree (in percentage) which was tracked during different biaxial shear tests. Breakage percentage represents the ratio of broken bonds to total number of bonds in each assembly. This diagram confirms that higher degree of breakage is achieved when larger value of confining pressure is applied in the simulations. In his triaxial tests on rockfill, Marsal (1973) showed that at the beginning of the test, larger particles that contain more flaws and defects, break indicating why the breakage rate at the beginning of the test is high. At the initial stage of the test, the smaller particles, produced by larger particle breakage and located in the voids among other intact large particles, do not play the role of transferring the force to their neighboring particles. After compaction of the assembly during the next stages, the gaps among particles become smaller and the small particles can play their role in transferring the force to their adjacent particles. Thus, the mean contact stress decreases owing to the increase of particle number surrounding each grain; therefore, the breakage rate will reduce afterwards. As shown in Fig. 4, the rate of particle breakage is high at the beginning of simulation and then slows down. Therefore, variation of breakage rate versus axial strain (and consequently axial stress) during the simulated biaxial test is in agreement with the trend observed by Marsal (1973).

In order to compare the results from these simulations with those from experimental tests, the experimental values of maximum principal stress ratio \( \frac{\sigma_2}{\sigma_1} \) provided by Gupta (2000), Venkatachalam (1993) and Marachi et al. (1972) together with the simulation tests results are shown in Fig. 5 against the degree of breakage \( (B_g) \). The degree of breakage is calculated from sieve analysis of rockfill sample as follows. Before testing, the sample is sieved using a set of standard sieves and the percentage of particles retained on each sieve is calculated. Due to particle breakage, the percentage of particles retained in large sieves will decrease and that in small size sieves will increase instead. The sum of decreases in percentage retained will equal that of increases. The sum of decreases (or increases) represents the value of the breakage factor \( (B_g) \) (Marsal, 1967). It is obvious that the simulation results are in
good agreement with experimental tests, which are close to the lower bound as shown in Fig. 5.

One way of investigating how the microstructure of a granular assembly evolves during the shearing process is to trace each particle displacement in the test. This becomes possible with numerical simulation in which the locations of all particles can be tracked. Fig. 6 represents movement trajectories of all particles in the unbroken test and in the test with breakable particles. In this sketch, the initial and final locations of all particles are connected to each other. It shows that all particles are trying to move towards the applied major and minor stress directions during the test but particles situated in the center of the assembly have the minimum movement during the test.

The microscopic evolutions of the assembly with non-breakable particles are referred to Mirghasemi et al. (1997), and those for the assembly with breakable particles to Seyedi Hosseininia (2004) and Seyedi Hosseininia and Mirghasemi (2006).

5. Effect of confining pressure

In the simulation tests, the effect of confining pressure can be observed, especially in test series A where the particles cannot break. The larger the confining pressure, the smaller the mobilized friction angle. In test series B, this trend exists to some extent but not for the whole range of applied strain during the test. The variation of volumetric strain against the axial strain is presented in Fig. 7. Since the confining pressure is high in all tests, the assemblies exhibit dense behavior and thus, all the samples dilate. However, the confining pressure does influence the dilatancy of the samples.

Some important trends are revealed in Fig. 7:

1. As $\sigma_{c.p.}$ increases, the peak value of the normalized stress decreases slightly. There is a small increase in the strain at which this peak occurs.
2. The normalized stress in the ultimate condition is more or less independent of $\sigma_{c.p.}$.
3. The volume increase is less for tests with larger confining pressure levels. In other words, dilatancy decreases with the increase in confining pressures. This is actually the case seen in the laboratory tests on soil with different confining pressures.

This behavior is attributed to two factors including confining pressure and interlocking. Interlocking increases as the confining pressure increases; therefore both the values of friction angle and dilatancy of assembly augment.

To figure out the effect of confining pressure on shear strength, one can sketch the Mohr circles for several tests to draw the Mohr–Coulomb failure envelope by drawing a line tangential to all circles, which should pass the origin of the diagram, because the particles in the assembly are assumed to be cohesionless. In Fig. 8, the Mohr envelopes for both tests are presented for five levels of confining pressure in terms of shear stress ($\tau = \sigma_2 - \sigma_1$) and normal stress ($\sigma = \sigma_1, \sigma_2$). The curvature of the envelope is generally true for granular soils tested using a wide range of confining pressures.

Having a curve tangential at the Mohr circles, the way in which a straight line can be fitted for a Mohr envelope will depend on the range of confining pressure ($\sigma_{c.p.}$). In a small range of confining pressure, the straight line passes through the origin indicating no cohesion; however, in large ranges of stresses, the line can be drawn in such a way that it shows some cohesion intercept for the whole media. The confining pressures selected for the simulation of biaxial tests in this research are relatively high;
therefore, it is expected to have some cohesion in the assembly, where the cohesion intercept in test A shows \( C' = 230 \, \text{kPa} \), but in test series B with breakable particles, the fitted line passes through the origin and cohesion intercept is zero.

Friction angles \( \phi_{\text{peak}} \) and \( \phi_{\text{residual}} \), as well as peak dilation rate \( \psi_{\text{max}} \) as a function of confining pressure can be investigated in Fig. 9. Frictional angles and peak dilation angle can be calculated from the following equations:

\[
\phi_{\text{peak (or residual)}} = \sin^{-1} \left( \frac{\sigma_2 - \sigma_1}{\sigma_2 + \sigma_1} \right)_{\text{peak (or residual)}}
\]

(2)

and

\[
\psi_{\text{max}} = \sin^{-1} \left( \frac{d \varepsilon_v}{d \varepsilon_l} \right)_{\text{max}}.
\]

(3)

In test A with non-breakable particles, the samples first reach a maximum friction angle (\( \phi_{\text{peak}} \)) and then go to an ultimate mobilized friction angle (\( \phi_r \)). These two values of friction angles are much larger than the mineral to mineral inter-particle friction angle (\( \phi_m \)). It is different in test series B with breakable particles, where there exists no peak mobilized friction angle and the samples directly go towards the residual mobilized friction angle (\( \phi_r \)). In both tests, the friction angles have a decreasing trend with increase of confining pressure except for the breakable assembly in the highest confining pressure of 8.0 MPa. It may be because most particles (about 94%) have been fragmented into several pieces and the new assembly with crushed and smaller particles shows new behavior and shear strength. It can be concluded that high degree of particle breakage may produce a new assembly with different resistance due to alteration of the soil particles.

Regarding the dilation of the assembly, it is obvious that maximum dilatancy angle (\( \psi_{\text{max}} \)) decreases with increase of confining pressure especially where particles are breakable. In these two simulation tests, we have found that \( \psi_{\text{max}} \) is less than inter-particle friction angle and mobilized friction angles, and in test B this reduction is much more obvious.
6. Comparison of the test results with the hyperbolic model

In this research, the stress–strain behavior of the assemblies has been compared with the hyperbolic elastic relationship suggested by Kondner and Zelasko (1963), who have stated that stress–strain curves of sand in standard triaxial compression test can be fitted by a hyperbolic equation of the form:

$$\sigma_1 - \sigma_3 = \frac{\varepsilon_1}{a + b \varepsilon_1},$$  \hspace{1cm} (4)

where constant $a$ is reciprocal of initial tangential Young’s modulus of the sample and $b$ is reciprocal of the ultimate deviatoric stress.

As the confining pressure increases, the modulus increases also with a power of $\sigma_{c.p.}$, for example as $E \propto \sigma^n$ with the exponent $n$ varying from 0.4 to 1.0. Larger exponent values are likely for loose soil samples. Janbu (1963) suggested the following expression to relate the confining pressure and modulus of elasticity:

$$E = K \cdot P_a \left( \frac{\sigma_{c.p.}}{P_a} \right)^n.$$ \hspace{1cm} (5)

To investigate if this relationship is governing in these two simulation tests, the results of all tests with confining pressure of 0.5, 1.0, 2.0, 4.0 and 8.0 MPa are considered. In logarithmic scale, five points of the aforementioned confining pressures are fitted by the best line. Fig. 10 shows the best fitted lines for both tests A and B with their equation and $R^2$-squared values. As it is clear, in both groups, the points are placed in a good way that a straight line is fitted among them with $R^2$ of 0.99. This means that Young’s modulus for various amounts of confining pressures can be estimated very well for this assembly.

The next step is to assess the values of $n$ and $K$ derived from these diagrams. The slope of the lines represents the value of $n$. As it can be seen, the $n$ value in breakable test is 0.83 which is larger than that of non-breakable test ($n=0.53$). This shows that modulus of the assembly with breakable particles is smaller than that of the assembly with non-breakable particles. It is likely that the breakable sample B may be treated as a looser medium than sample A. It is worth noting that the intercept of this diagram denotes $\log K$. The final form of modulus relationships are as follows:

$$E_i = 10^{2.1} P_a \left( \frac{\sigma_{c.p.}}{P_a} \right)^{0.83}, \quad \text{for breakable assembly; \hspace{1cm} (6)}$$

and

$$E_i = 10^{3.1} P_a \left( \frac{\sigma_{c.p.}}{P_a} \right)^{0.53}, \quad \text{for non-breakable assembly. \hspace{1cm} (7)}$$

Comparing Eqs. (6) and (7) reveals that $K$ factor for breakable assembly ($K=10^{2.1}=125.9$) is about 1/10th of that for non-breakable assembly ($K=10^{3.1}=1259$). By dividing these two equations, we obtain:

$$\frac{E_{i,\text{breakable}}}{E_{i,\text{non-breakable}}} = 0.1 \left( \frac{\sigma_{c.p.}}{P_a} \right)^{0.3}.$$ \hspace{1cm} (8)

Eq. (8) shows the relation of deformation modulus of an assembly in two modes where the particles can break or cannot break. This relationship is not a complete one because it does not include the degree of breakage of the particles.

Fig. 10. Determination of parameters $K$ and $n$ to estimate the initial Young’s modulus with Janbu formula (1963).
7. Effect of rock strength on the behavior of the assembly

Three biaxial compression tests have been simulated with different rockfill materials to investigate the effect of rock strength on the behavior of the assembly. Rock strength parameters are shown in Table 1. All these tests are performed in the same condition with confining pressure of 2.0 MPa. The inter-particle frictional coefficient is held constant and equal to 0.5. Fig. 11 shows the variation of $\sin(\phi_{mobilized})$, volumetric strain and degree of total bond breakage (%) against the axial strain during the tests.

As illustrated in Fig. 11, although the amount of breakage is reduced with increasing rock strength, there is little sense of difference in their corresponding shear strength $\sin(\phi_{mobilized})$. This may be ascribed to high amount of breakage in all tests and almost the same level of breakage degree (75–92% at the end). The difference in behavior will be observed more clearly if the difference of breakage degree is in a larger range, noting that the assembly with highest rock strength (sample #3) shows a reduction in the mobilized $\sin \phi$ at the large axial strain. However, it behaves almost the same as other samples at the beginning and the middle of the test.

The other point which can be found out in this series of tests is the relation between shear strength of the assembly and its dilation: the higher the assembly dilates, the higher the shear strength becomes. This point was also revealed when the assembly was studied in different confining pressures.

8. Conclusions

Comparison of two simulated series of biaxial tests indicated that particle breakage leads to a decrease in internal angle of friction and an increase in granular material compressibility. Also the rate of particle breakage in different confining pressures during biaxial tests was investigated. The results are similar to data obtained from experimental tests on real rockfill materials.

The proposed hyperbolic elastic formula for sand was investigated for both groups of assemblies and it was observed that in both situations the results are in good agreements with those measured in experimental tests.

### Table 1

<table>
<thead>
<tr>
<th>Sample #</th>
<th>Compressive strength (MN/m²)</th>
<th>Tensile strength (MN/m²)</th>
<th>Interceptor (MN/m²)</th>
<th>Coefficient of static friction (tan $\phi$)</th>
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<td>1</td>
<td>190</td>
<td>19</td>
<td>40</td>
<td>1.6</td>
</tr>
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<td>2</td>
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<td>35</td>
<td>75</td>
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By changing the strength of particle breakage in a parametric study, it was seen that the amount of breakage has a reverse relation with increasing rock strength. Also, the higher the assembly dilates, the higher the shear strength becomes.

Comparison between simulation results and observations from experimental tests shows that the method presented for modeling breakage can help us to have a qualitative view about the effect of breakage phenomenon on behavior of granular materials.

References


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