Numerical simulation of breakage of two-dimensional polygon-shaped particles using discrete element method

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Abstract

Using DEM (Discrete Element Method), a model is presented to simulate the breakage of two-dimensional polygon-shaped particles. In this model each uniform (uncracked) particle is replaced with smaller inter-connected sub-particles which are bonded with each other. If the bond between these sub-particles breaks, breakage will happen. With the help of this model, it is possible to study the influence of particle breakage on macro and micro mechanical parameters. In this simulation, the evolution of microstructure in granular assemblies can be seen by tracing of coordination number during the shear process. Also variation of contact normal, normal force and tangential force anisotropy can be tracked. To do so, two series of biaxial test simulations (breakage is enabled and disabled) are conducted on assemblies of two-dimensional polygon-shaped particles and the results are compared. The results are presented in terms of macro and micro mechanical behavior for different confining pressures.

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1. Introduction

Stability of soil structures such as breakwaters and rockfill dams is in concern of shear parameters and the behavior of granular media. In general, shear resistance and behavior of granular materials depends on different factors such as mineralogical composition, particle grading, size and shape of particles, fragmentations of particles and stress conditions.

Breakage of particles will happen in such high structures especially in the lower layers where there are levels of significant pressure caused by the upper layers. Crushing of large particles into smaller ones result in changes in the design gradation curve; therefore, the mechanical behavior of granular material alters.

Influence of particle breakage on internal friction angle and deformability of granular materials can be studied using experimental tests such as triaxial and unconfined compression tests [9,1,4,8,27]. Performing triaxial compression tests on coarse granular materials, Marsal [9] found out that the most important factor affecting both shear strength and compressibility is the phenomenon of fragmentation undergone by a granular body when subjected to change in its state of stress both during uniform compression stage and during deviatoric load application. Also the results showed that in granular media, the compressibility is a consequence of complex phenomenon that takes place as a result of displacements between particles combined with the particle breakage. Varadarajan et al. [27] have investigated the behavior of two dam site rock materials (Ranjit Sagar and Purulia) in triaxial compression tests which the former consisted of rounded and the latter angular particles. It is interesting to note that the volume change behavior of two rockfill materials was significantly different from each other. During the shearing stage of the triaxial test, compression, rearrangement and breakage of particles took place. The rounded material exhibited continuous volume compression, while the angular particles dilated and expanded after initial compression in volume; angular materials provide a high degree of interlocking and cause dilation during shearing. Also they observed that a greater degree of particle breakage occurs with the larger particles because of the greater force per contact [7]. There are two factors governing on the shear resistance as interlocking between particle and particle breakage. The effect of increase in interlocking is to increase the shearing resistance, while the effect of breakage of particles is vice versa. Also it is noted that
angular particles are more susceptible to break than rounded particles.

Performing such tests on material with large particles in order to study the behavior of materials such as rockfill would be costly due to the required large size of specimen. At the present research, an investigation is made to study the influence of particle breakage on behavior of granular media using a numerical method. In recent years, along with the progress of numerical methods and computer technology, different methods have been used to model breakage of brittle bodies with the help of Discrete Element Method (DEM). Among these methods, are the approach used by Cundall [3], Potapov and Campbell [17,18], the method based on simultaneous utilization of Molecular Dynamics (MD) (Kun and Herrmann [6]), and the 3D approach used by Robertson and Bolton [19], McDowell and Harireche [11].

2. Particle breakage simulation in DEM

Prior to description of the method used at the present research, a brief overview of the above methods is presented.

2.1. Brief review of different modelings

Cundall who is a pioneer in use of DEM in studying behavior of granular media and stability of rock slopes prepared a code called RBMC in which breakage mechanism of rock blocks was simulated similar to that of Brazilian test [3,28]. In this code, in each cycle of simulation all of the point loads applied to each block are checked and then the application point and magnitude of the two maximum loads, which are applied in opposite direction of each other, are determined.

RBMC is based on the conception that particle breakage happens instantly (during 1 cycle) and the block is divided into two pieces through the line connecting application point of the loads. In this method, after each breakage occurrence, the primitive block is omitted and two blocks with new geometry are generated; thus it is necessary to calculate the geometry, mass and moment of inertia of these new particles and save them in a big amount of computer memory.

The other procedure for breakage simulation is the method applied by Potapov and Campbell [17,18]. They have studied the breakage induced in a single circular particle that impacts on the solid plates (1994) and brittle particle attrition in a shear cell (1997). They tried to model the breakage of square particles placed between two top and bottom plates with 90° saw tooth patterns. In both simulations, a breakable solid material is created by gluing together unbreakable and non-deformable solid triangular elements. It is assumed that a glued joint can only withstand normal force.
tensile stress up to some limit. If the tensile stress on any portion of the joint exceeds the limit, the glue along that portion breaks and can no longer support tensile stress; crack forms but only along that portion of the joint for which the tensile strength is exceeded.

The other study in the area of breakage modeling and crack formation in brittle bodies are based on simultaneous use of DEM and MD (Kun and Herrmann [6]). Molecular Dynamics (MD) is a computational technique that considers a macroscopic material as an assemblage of microscopic particles.

In order to study the process of fragmentation in two-dimensional brittle blocks and observing the relationship of the size of broken parts with one another, Kun and Herrmann [6] considered each block as a mesh of inter-connected tiny cells located in a plane. This cellular mesh is generated by the use of a random process (Voronoi Construction). Each cell is a rigid convex polygon that as the smallest component of the block neither breaks nor deforms and acts as a distinct element of other cells. Cells have one rotational and two linear degrees of freedom in the block plane and their behavior in contact is simulated by DEM.

In order to keep the unity of the cells forming a block, the center of mass (center of area) of each cell is connected to the mass center of neighboring cells through an elastic beam. If at a specific time during the simulation, the relative displacement between two cells enlarges so much that the stress formed in the beam connecting them exceeds the beam bearing capacity, the bond will be broken. This is the starting point in crack formation and the crack enlarges gradually as the beams connecting the consequent particles break. When an assemblage of inter-connected cells is thoroughly disconnected from the primitive block, breakage has occurred. In the recent model, it is feasible to study the process of crack formation in a brittle body and not in an assembly of particles that each one behaves as an individual body and they have no connection to each other.

In order to simulate three-dimensional crushable soils, an approach is produced using DEM. In this method, agglomerates are made by bonding elementary spheres in ‘crystallographic’ arrays. This approach is used in the program PFC3D (Itasca Consulting Group, 1999). This program uses the soft contact approach of the distinct element methods, which assumes that each element has a finite normal stiffness and represents elastic flattening at contacts allowing the bodies to overlap. A stiffness model, a bonding model and slip model are included in the constitutive representation of contact points between the elementary spheres that are the basic building block. In the linear contact model, it is assumed that each sphere have a normal and a shear stiffness. The simple contact bond can be envisaged as a pair of elastic springs at a point of glue. It serves to limit the total normal and shear forces that the contact can carry by enforcing bond-strength limits. The maximum tensile force that the bond can sustain in tension and the maximum shear force it can withstand before breaking are specified when the bond is created and may be modified at any time during the simulation. The bond breaks if either of these values is exceeded. As the simple contact bond acts over a vanishing small area of contact point, it does not resist bending moment. This means that it has no resistance to rolling of a sphere bonded adjacent to it if no third body exists to restrain the motion. This approach has been used by Robertson and Bolton [19] and McDowell and Harireche [11].

A slip model acts between unbonded objects in contact, or between bonded objects when their contact breaks. It limits the shear force between objects in contact and allows slip to occur at a limiting shear force, governed by Coulomb’s equation.

This approach has been accomplished for simulation of silica sand grains and the results compared with the real data for silica sand. [2] Although this method can model the behavior of sands well, it can not be used for the particles with sharp angles such as rockfills, since the sand agglomerate in this method consists of smaller rounded spheres.

2.2. Breakage modeling in this research

The aim of this research is to find the influence of breakage of angular shaped particles on the behavior of materials such as rockfill. It has been tried to model the particle breakage in a way that less number of particles and computational effort be needed [12,16]. To achieve this, the particles are studied in a two-dimensional space.

In the present research, simulation of biaxial test is performed on assemblies of 500 particles within 1500 sub-particles using personal computer (PC). For this purpose, the program POLY [13,14] is developed to model assemblies of irregularly shaped particles with the ability of breakage [15,16,23].

It is assumed that each rockfill particle can break through straight lines with certain direction and position. The lines are determined in a way that two commonly observed behavior in fragmentation can be simulated. These two kinds of behavior are crushing of particle vertexes and cracking across a particle that divides particle into pieces. The lines are predefined and are randomly distributed within the particles. According to Fig. 1(a), it is assumed that particle P can only break through the lines d1, d2 and d3. Thus in this method, each uncracked particle (P) consists of smaller bonded particles like P1, P2,...and Pn. Particle P is called Base Particle and particles P1 to Pn are called Sub-Particles. The
sub-particles are considered to be rigid bodies. They are not breakable and not deformable. But they can overlap when they are pressed against each other. The base particles are not deformable either, but they are breakable since they are made up of several sub-particles. The both base and sub-particles are arbitrarily convex polygon-shaped.

In order to ensure the rigidity and continuity of the bonded sub-particles to form a base particle, it is assumed that two adjacent sub-particles are connected with a fixed connection at the middle of their common edge (points m1 and m2 in Fig. 1(b)). This fixed connection plays the role of the bond between two bonded sub-particles. During the simulation, when the existing bond stress exceeds its final bearing capacity, the connection will break and with separation of the two bonded particles, breakage takes place.

For modeling the connection between two bonded particles, two transitional and one rotational springs are introduced. One of the transitional springs that are perpendicular to the common face of particles is normal spring and the other one which is parallel to the common edge is the shear spring. Moment and forces at the connection between two bonded particles are transferred through rotational and transitional spring, respectively. They can be calculated according to relative displacement of two sub-particles at each simulation cycle.

Fig. 2 illustrates a base particle P in an assembly of particles subjected to an arbitrary loading. Due to interaction between particles, the forces and moments are induced at base particle’s contact points with adjacent particles. As a consequence, the sub-particles P1 and P2 are relatively displaced against each other. Therefore, the points m1 and m2 are no longer coincident. To determine the force and moment applied on each contact edge between two sub-particles, the relative displacement of the two sub-particles P1 and P2 is replaced with three components, $\Delta_n$ (normal displacement), $\Delta_s$ (shear displacement) and $\Delta_\theta$ (rotational displacement) (Fig. 2). Hence, the normal and shear forces and the moment at the contact point can be expressed as follows:

\[ F_{n\text{-Bond}} = K_{n\text{-Bond}} \Delta_n \]
\[ F_{s\text{-Bond}} = K_{s\text{-Bond}} \Delta_s \]
\[ M_{\text{Bond}} = K_{\theta\text{-Bond}} \Delta_\theta \]

where $K_{\theta\text{-Bond}}$ is the stiffness of the rotational spring and $K_{n\text{-Bond}}$ and $K_{s\text{-Bond}}$ are unit length stiffness of the normal and shear springs, respectively. Values of these parameters are considered to be proportional to the stiffness of the particles.

With the existence of one of the following conditions, the bond between the two adjacent sub-particles is broken and separation happens:

1. If the shear stress at the bond between the two particles is larger than the allowable shear stress (the bond bearing capacity).
2. If the maximum compressive or tensile stresses caused by moment and normal force of the bond exceeds the allowable compressive and tensile stresses.

In other words, despite of the model presented by Potapov and Campbell [17,18], the bond will be failed due to three modes of compression, tension or shearing. The bond bearing capacity obeys from the Coulomb failure criterion for rocks which is extended in both compressive and tensile stresses, but they are limited by magnitudes of stresses obtained from unconfined compressive strength and Brazilian tensile strength tests respectively.

The same procedure of modeling would be suggested in 3-D computation. The base particles could be 3-D multi-surface blocks that are made up of several bounded 3-D blocks. The sub-particles would be 3-D blocks that have connecting faces in touch. These

Table 2
<table>
<thead>
<tr>
<th>Parameters used test B (Breakage enabled)</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Normal and tangential stiffness (N/m)</td>
<td>$2.0 \times 10^7$</td>
</tr>
<tr>
<td>Unit weight of particles (kN/m³)</td>
<td>2500</td>
</tr>
<tr>
<td>Time step (s)</td>
<td>1.52E−4</td>
</tr>
<tr>
<td>Strain rate</td>
<td>0.005</td>
</tr>
<tr>
<td>Module of elasticity (E) (MN/m²)</td>
<td>$9.0 \times 10^6$</td>
</tr>
<tr>
<td>Rock strength parameters</td>
<td></td>
</tr>
<tr>
<td>Compressive strength (MN/m²)</td>
<td>350</td>
</tr>
<tr>
<td>Tensile strength (MN/m²)</td>
<td>35</td>
</tr>
<tr>
<td>Intercept (cohesion) (MN/m²)</td>
<td>75</td>
</tr>
<tr>
<td>Coefficient of static friction</td>
<td>1.60</td>
</tr>
</tbody>
</table>

![Fig. 5. (a)Relationship between sin $\phi$ and axial strain (b)Relationship between volumetric strain and axial strain ($P_{cp} = 2.0$ MPa).](image-url)
faces act actually as the planes of resistance against compression, tension and shearing.

2.3. Biaxial simulation

Simulation of two series of biaxial compression tests is fulfilled with four levels of confining pressures ($P_{c.p.}$) of 0.5, 1.0, 2.0 and 4.0 MPa to investigate the particle breakage in a granular media. In the series test A particles are rigid with no ability in fragmentation while in the series test B the particles are breakable. The particles are placed in a circular area.

Each test includes three stages. At first, the initial computer assembly of particles is generated, then subjected to a defined confining pressure and finally the assembly is sheared biaxially at a constant deviatoric strain rate (along the $2-2$ axis). The assembly of particles at different stages (breakage enabled) is presented in Fig. 3. Also, two different areas of the whole assembly are magnified to show more clearly the breakage of particles. Fig. 4 shows the growth of bond breakages during hydrostatic compaction and shearing processes, respectively. The inter-particle friction coefficient is set to 0.5 for all tests and the particles are assumed to be cohesionless at the contact. Also the particles are considered weightless. The parameters used for tests A and B are summarized in Tables 1 and 2, respectively.

3. Results

The results obtained from the simulations can be studied from two different points of view as macro and micromechanical considerations.

3.1. Macromechanical observations

The results of biaxial simulations in tests A and B for $P_{c.p.} = 2.0$ MPa are presented in the form of curves of $\sin \phi_{\text{mobilized}}$ (Eq. (2)) versus axial strain and volumetric strain versus axial strain (Fig. 5).

As shown in both tests, the shear strength ($\sin \phi_{\text{mobilized}}$) increases and then reaches to a constant value. In test A, the $\sin \phi_{\text{mobilized}}$ grows rapidly and reaches at a peak of 0.6, but in test B, it gradually increases and it becomes constant just under 0.5. It seems that particle breakage has a decreasing effect on shearing resistance of the assembly.

$$\sin \phi_{\text{mobilized}} = \frac{\sigma_2 - \sigma_1}{\sigma_2 + \sigma_1} = \frac{\sigma_2}{\sigma_1} - \frac{1}{\sigma_2 + \sigma_1} \quad (2)$$

In general, the angular particles have dilative behavior (Marsal [9]). Fig. 5(b) shows that the assembly with no breakage has a more dilative behavior than that with the ability of fragmentation. As investigated before, the more the assembly dilates, the larger is the shear resistance. In test B, particles can not undergo the forces imposed on them and breakage happens, therefore smaller particles fill the voids and let the other particles move freely. This causes the assembly to show a compressive behavior in larger axial strains followed by increasing volumetric strain. This trend can justify the reduction of $\sin \phi_{\text{mobilized}}$ in test B. The same result has been obtained in experimental test results [9,4].

Fig. 6 illustrates the variation of bond breakage in three modes of failure which have been tracked during biaxial shear test with confining pressure of 2.0 MPa. The breakage percentage shows the ratio of broken bonds to total number of bonds. It should be noted that initial amounts of breakage shown in the figure, pertains to the breakage of particles resulted in the isotropic compression stage. The possible causes of fragmentations are due...
to compression, tension and shear breakage. It shows that no particle breakage has happened due to compression. Although in hydrostatic compression stage, most particles have been broken because of shear failure (35%), in biaxial test the degree of tension breakage is higher which started from about 13% at the beginning and it reached at just below 40%. Also the number of shear breakage is to some extent constant at large axial strain whereas the tension breakage grows gradually.

Having performed triaxial tests on rockfill, Marsal [10] showed that at the beginning of the test, larger particles that contain more flaws and defects, break and it is why the breakage rate at the beginning of the test is high. At the primitive stages of the test, the smaller particles, produced by larger particles breakage, are located in the voids between the other intact large particles and consequently have no role in transferring the force to their neighboring particles. After compaction of assembly during next stages, the gaps between particles become smaller and the small particles can play their role in transferring the force to the adjacent particles. Thus the mean contact stresses decrease owing to the increase of particles surrounding each grain; therefore, the breakage quantity will reduce afterwards. Considering the total number of breakage in Fig. 6, the rate of particle breakage is high at the beginning of simulation and then it slows down. Therefore variation of breakage rate versus axial strain (and consequently axial stress) during the simulated biaxial test is in agreement with the trend observed by Marsal [10]. The same results can be found out in DEM simulation [24].

The effect of confining pressure on shear strength is in reverse where the higher pressure results in the lower mobilized friction angle in both tests (Fig. 7). Also the axial strain corresponding to the maximum shear strength values increases with increasing confining pressure.

In Fig. 8, it can be seen that assembly with no breakage has a more dilative behavior than that with the ability of fragmentation. The higher confining pressure on the specimen causes to compress it more and does not let the sample dilate. On the other hand, under higher pressures particles have more tendencies to be broken. This causes the assembly to show a more compressive behavior under larger confining pressures. This trend can describe the reason for the reduction of $\sin \phi_{\text{mobilized}}$. On overall, for both series of simulations, the more the assembly dilates, the larger the shear resistance is. The same result has been obtained in experimental test results [9,4,27].

Table 3 shows obtained frictional angle of the assembly in different confining pressures. Also Fig. 9 illustrates the variation of bond breakage degree (in percent) which has been tracked during different biaxial shear tests. This diagram confirms that higher degree of breakage is achieved when the larger value of confining pressure is used in the simulations.

All the simulations presented here have been carried out in 2-D state. Thus, it is difficult to compare quantitatively the obtained results with the 3-D experimental tests such as triaxial compression. However it is possible to compare the trend of results influenced by different parameters. In order to compare

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Comparison of internal friction angle in simulated tests</th>
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<tbody>
<tr>
<td>Confining pressure</td>
<td>With no breakage</td>
</tr>
<tr>
<td>0.5 MPa</td>
<td>37.6°</td>
</tr>
<tr>
<td>1.0 MPa</td>
<td>36.2°</td>
</tr>
<tr>
<td>2.0 MPa</td>
<td>34.8°</td>
</tr>
<tr>
<td>4.0 MPa</td>
<td>33.4°</td>
</tr>
</tbody>
</table>

Fig. 8. Volumetric strain versus axial strain during the tests.

Fig. 9. Variation of bond breakage with axial strain in different confining pressures.
qualitatively the obtained results from these simulations with the experimental tests results, the values of the maximum principal stress ratio \( \sigma_2 / \sigma_1 \) in the biaxial simulation tests and the experimental tests performed by Gupta [5], Venkatachalam [26], Marachi et al. [8] are shown in Fig. 10 versus the degree of breakage (\( B_g \)). The value of breakage is calculated from sieve analysis of rockfill sample as follows. Before testing, the sample is sieved using a set of standard sieves and the percentage of particles retained in each sieve is calculated. Due to the breakage of particles, the percentage of particles retained in large size sieves will decrease and the percentage of particles retained in small size sieves will increase. The sum of decreases in percentage retained will be equal to the sum of increases in percentage retained. The sum of decreases (or increases) is the value of the breakage factor (\( B_g \)) [9]. It is observed that the simulation results fall inside the lower bound shown in the figure. As shown in Fig. 10, the degree of breakage increases with decrease of ratio \( \sigma_2 / \sigma_1 \) in both simulation and experimental tests. If the ratio \( \sigma_2 / \sigma_1 \) can be interpreted as the strength of the assembly, the observed trend is logical and the simulation and experimental results are qualitatively in agreement.

### 3.2. Microscopic behavior

While it is obvious that forces in granular media must be carried by means of contacts between particles, it is only recently that a means of quantifying the arrangement of contacts has been developed. For any angle \( \theta \), the portion of the total number of contacts in the system that are oriented at angle \( \theta \) is \( E(\theta) \). The distribution of contact normal orientations is described by a function such that the fraction of all assembly contact normals...
falls within the orientation interval $\Delta \theta$. Rothenburg et al. (1989) [20] showed that the distribution of such contacts takes the form:

$$E(\theta) = \frac{1}{2\pi} \left[ 1 + a \cos(\theta - \theta_0) \right]$$  \hspace{1cm} (3)

where $a$ is referred to as the parameter of anisotropy, and $\theta_0$ is the major principal direction of anisotropy. The meaning of $a$ becomes clear if it is noted that the number of contacts oriented along the direction of anisotropy, i.e. when $\theta = \theta_0$ is proportional to $1 + a$ while the number of contacts oriented in the perpendicular direction is proportional to $1 - a$. The parameter $a$, therefore, is proportional to the difference in the number of contacts oriented along the direction of anisotropy and in perpendicular direction. A similar expression was introduced by Thornton and Barnes [25].

The magnitudes of the contact forces in an assembly with irregular geometry vary from contact to contact. Despite the apparent randomness in the variation of contact forces, regular trends emerge when they are averaged over groups of contacts with similar orientations. The average contact force acting at contacts with an orientation can be decomposed into an average normal force component, $\bar{f}_n^c(\theta)$, and an average tangential force component, $\bar{f}_t^c(\theta)$. By averaging the contact forces of the contacts falling within the group of similar orientation and following the same logic as for the contact normals, symmetrical second-order tensors may be introduced to describe average normal contact forces and average tangential contact forces. The average normal contact force tensor can be defined as [20]:

$$\bar{f}_n^0(\theta) = f_n^0 [1 + a_n \cos(\theta - \theta_f)]$$  \hspace{1cm} (4)

where $a_n$ is the coefficient of normal force anisotropy, and $\theta_f$ is the major principal direction of force anisotropy; $f_n^0(\theta)$ is the average normal contact force from all assembly contacts.

The average tangential contact force tensor can be defined as:

$$\bar{f}_t^0(\theta) = f_t^0 [a_t \sin(\theta - \theta_0)]$$  \hspace{1cm} (5)

where $a_t$ is the coefficient of tangential force anisotropy and $\theta_0$ the direction of anisotropy.

The general expression for the average stress tensor can now be written as:

$$\sigma_{ij} = m_c l_0 \int_0^{2\pi} \left[ \bar{f}_n^c(\theta) n_i n_j + \bar{f}_t^c(\theta) t_i t_j \right] E(\theta) d\theta$$  \hspace{1cm} (6)

where $m_c$ is the average number of contacts per unit area (volume), $l_0$ is the assembly average contact vector length.

Fig. 12. Displacement trajectories of all particles during the biaxial test in (a) unbreakable test; (b) breakable test.

Fig. 13. Coordination number’s variation with axial strain ($P_{cp} = 2.0$ MPa).
(average distance from the particle centers to the contact point), \( n_c \) is the contact normal vector, and \( r_c \) is the contact tangent vector. Rothenburg and Bathurst [20–22] derived a relationship between the measure of shear stress and the parameters \( a, a_n, a_t \) involved in the characterization of anisotropies in contact orientations and contact forces according to Eqs. (2)–(5). For the case when the directions of anisotropy in contact forces and contact orientations coincide, as in a biaxial test, the relationship is as follows:

\[
\frac{\sigma_2 - \sigma_1}{\sigma_2 + \sigma_1} = \frac{a + a_n + a_t}{2 + a_n \times a_t}
\] (7)

The simplified expression suggests that the capacity of a cohesionless granular assembly is directly attributable to its ability to develop anisotropy in contact orientations or to withstand directional variations of average contact forces. Eq. (7) was evaluated for assemblies of disc-shaped and elliptical particles [20]. The application of this theory is also evaluated for polygon-shaped particles [13]. So the same model can be applicable to study the microscopic behavior of breakable particles. For this purpose, each base particle that has not been yet broken into pieces is considered as one particle, but the sub-particles are considered as individual new particles as soon as they are separated from their base particles. In this condition, during the simulation the number of particles is increased and accordingly all micro parameters are being averaged in each step for the current number of individual particles.

One can find out the evolution of the anisotropy coefficients along the increment of axial strain by drawing them against each other. Also at each axial strain, one can sketch the anisotropy state of the assembly with a histogram which shows a polar distribution of the anisotropy. If there is no anisotropy in an assembly, the histogram shape should be close to a circle, which implies that the parameter magnitude is the same in all direction, but with the growth of anisotropy, the shape grows to be deformed such as a peanut. The histogram shape of anisotropy coefficient of tangential force \( (a_t) \) is different from the histogram shape of contact normal \( (a_n) \) and normal force \( (a) \) coefficients which are like a peanut. Apart from measuring the anisotropy of the parameters and sketching the histograms, fitted histograms can be obtained from the tensor functions stated above by having the coefficient and the principal angle of anisotropy. Fig. 11 shows three histograms of different anisotropy coefficients in the beginning (axial strain=2.0%) and at the end (axial strain=16.0%) of a biaxial test with breakable particles at \( P_{c.p.} = 2.0 \) MPa.

One way of investigating how a microstructure of granular assembly evolves during the shearing process is to trace each particle displacements along the test. This is possible to see with numerical simulation in which the locations of all particles can be under controlled. Fig. 12 represents the movement of all the particles in the unbroken test and all the sub particle displacements in the test with breakable particles. In this sketch, the initial and final locations of all particles are connected to each other. It shows that all particles are trying to move towards the implied major and minor stresses during the test whereas the particles situated in the center of the assembly have the minimum movement during the test.

The other way of investigating the evolution of microstructure of granular assembly is to study the change in the number of contacts in the assembly or the average coordination number of the system.
Fig. 13 presents the evolution of the average coordination number during shear deformation at $P_{c.p.}=2.0$ MPa. At the beginning, the coordination numbers of two assemblies are different owing to the hydrostatic compression stage carried out previously. But the way of creating contacts is different in these two series of tests. In tests B, the particles can be broken into pieces and then each new particle makes new contacts with its neighbor particles, while this phenomenon does not happen for the particles in tests A. Therefore, both the number of particles and the number of contacts during the hydrostatic loading are increasing. Consequently, the coordination number at the final stage of compaction will differ from that of assembly with non-broken particles. As can be seen, the growth of the numbers of particles and contacts due to bond breakages in test series B (with breakage), is in a way that at the beginning of shear stage, the coordination number is less than that of the tests A (with no breakage). Also the trends of coordination number variation are different during shearing process. As shown in Fig. 14, in test series A, it decreases rapidly along axial strain and comes to be constant, while in test B, it grows gradually towards a constant value in high confining pressures. During each test of both series of tests, contacts in the assembly began to degrade as the axial stress increased, mainly in the horizontal direction. But in test series B, particle breakage is also happening at the same time and this phenomenon results more development of contacts between particles. That is why the number of contacts grows. The effect of increasing the confining pressure on coordination number can be observed from the figures. The higher confining pressure, the more contacts are induced in the assembly [24]. As mentioned before, the initial values of the coordination number are because of the isotropic compaction stage before the assembly is sheared biaxially.

The variation of the contact normal anisotropy (parameter $a$) as a function of axial strain is illustrated in Fig. 15. It describes the degree of anisotropy in contact orientations. This coefficient is essentially proportional to the difference in the number of contacts in each direction, and describes the degree of anisotropy in contact orientations.

The coefficient of fabric anisotropy evolves to the maximum values as contacts are lost, mostly oriented along the direction of tensile strain (horizontal direction). But this growth is more rapid in tests A than in test B where particles can break. Also in test A, the coefficient reduces to the lower ultimate values at large axial strain, but in test B, it continuously and slowly grows to a peak value of half of that in test A. In Fig. 16, the effect of confining pressure on this parameter can be investigated. The confining pressure has a reverse effect on this coefficient, but this rule is not always correct in the test series B.
Fig. 17 presents the development of anisotropy in normal contact forces for $P_{c,p}=2.0$ MPa by plotting the variation of the coefficient of normal contact force, $a_n$, with axial strain during the simulations on assemblies with rigid and breakable particles. In both tests, by applying deviatoric axial strain in the vertical direction, the normal forces carried by chains of particles in the vertical direction are increased, while the magnitude of average normal forces in the horizontal direction remains almost constant. The parameter $a_n$ indicates the difference in the values of average forces on vertical and horizontal contacts. A greater difference in the magnitude of the average forces in the horizontal and vertical directions provides a higher value for $a_n$. In test A, as the axial strain increases, $a_n$ shows a rapid growth at lower axial strain, followed by a reduction after the maximum value. This is because of loss of contacts and also the loss of the capacity of chains of particles to sustain high forces. In contrast to test A, when the particles can be divided into smaller ones, the parameter $a_n$ shows a gradual increase which reaches at a constant value. This behavior of $a_n$ is sensitive while the particles cannot tolerate the imposed forces and breakage happens, therefore, particles can not make a chain to show a peak in $a_n$. Although the increasing of $a_n$ are in different manners in these two tests, it might be anticipated that $a_n$ reaches to the same ultimate value at large axial strain.

The coefficient of tangential contact force anisotropy, $a_t$, shows a rapid rise at small axial strain. In test A, the coefficient of tangential force anisotropy reaches to the maximum value, followed by a slow reduction in magnitude, as illustrated in Fig. 18. In spite of test A, $a_t$ in test B continues to grow very slowly to a constant value. The initial increase in $a_t$ is due to the development of frictional resistance as a result of potential relative movement between adjacent particles. As the number of contacts reduces, the particles gain more opportunity to rotate; therefore in test A, tangential forces are slowly released. In test B, particles

![Fig. 19. Evolution of force anisotropy coefficient.](image)

![Fig. 20. Verification of relationship between stress and fabric for assembly with and without breakable particles ($P_{c,p}=2.0$ MPa).](image)
cannot bear the tangential forces and get into pieces so they have the opportunity to move freely in the voids between other particles. Particle breakage causes not to mobilize the shear forces completely; therefore shear force does not reach to a peak. Also, it can be observed that assemblies with lower confining pressure can provide more anisotropy in the media with subsequent higher shear strength. Fig. 19 shows the variation of anisotropy coefficients with different confining pressures [24].

Table 4

<table>
<thead>
<tr>
<th>Strength parameters</th>
<th>Sample #1</th>
<th>Sample #2</th>
<th>Sample #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength (MN/m²)</td>
<td>190</td>
<td>350</td>
<td>650</td>
</tr>
<tr>
<td>Tensile strength (MN/m²)</td>
<td>19</td>
<td>35</td>
<td>65</td>
</tr>
<tr>
<td>Intercept (MN/m²)</td>
<td>40</td>
<td>75</td>
<td>140</td>
</tr>
<tr>
<td>Coefficient of static friction (tan(ϕ))</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The coefficients of anisotropy $a_n$ and $a_t$ were substituted into the stress–force–fabric relationship (Eq. (7)). Fig. 20 compares the computed shear resistance based on theory (right-hand side of Eq. (7)) and the value of sin $ϕ_{mobilized}$ (left-hand side of Eq. (7)) provided by biaxial simulations on assemblies with angular particles for both tests A and B. As can be observed, the stress–force–fabric expression is in agreement with the measured shear resistance in the test simulations.

4. Effect of rock strength on the behavior of the assembly

Three biaxial compression tests have been simulated with different rockfill materials to investigate the effect of strength on the behavior of the assembly. The strength parameters are shown in Table 4. All these tests are performed in the same condition with confining pressure of 2.0 MPa. The inter particle frictional coefficient is held constant and equal to 0.5. Fig. 21 shows the variation of sin $ϕ_{mobilized}$, volumetric strain and degree of total bond breakage (%) along the axial strain during the tests.

As illustrated in Fig. 21, although the amount of breakage is reduced with increasing rock strength, there is little sense of difference in their corresponding shear strength (sin $ϕ_{mobilized}$). Perhaps this is because of high amount of breakage in all tests and

![Fig. 21. Variation of (a) sin $ϕ_{mobilized}$; (b) volumetric strain; (c) total breakage degree; with axial strain in confining pressure of 2.0 MPa for different rock strengths.](image-url)
also the same level (75%–92% at the end) of the degrees of bond breakage. The differences in behavior would be probably observed more clearly if the variation of breakage degree were in a larger range.

The other point which can be found out in this series of tests is the relation between the shear strength of the assembly and the dilatation; the higher the assembly dilates, the higher the shear strength becomes. This point was also gained when the assembly was studied in different confining pressures.

5. Conclusions

The results of two simulated series of biaxial tests with several confining pressures indicated that higher confining pressure leads to decrease in shear strength and increases granular material compressibility. In return, the dilatancy falls down. This shows that the higher the dilatancy is, the higher the shear strength becomes. The same result was gained with a series tests with different rock strengths. Also the rate of particle breakage in different modes was investigated. The higher the confining pressure, the more the degree of breakage is. The results are similar to data obtained from experimental tests on real rockfill materials.

The influence of confining pressure on the variation of micromechanical parameters was also studied. The assembly in which breakage is enabled, the coordination number remains almost constant during the shearing test, but in the other group, it decreases along with axial strain. As observed, the magnitude of normal contact force and tangential force anisotropy coefficients are smaller in the case of breakable particles than those in rigid particles. But the confining pressure has a reverse effect on the anisotropy coefficients. The shear strength of granular assembly is directly attributable to the ability to develop anisotropy.

The stress–force–fabric expression developed by Rothenburg and Bathurst [20,21] was verified for the assemblies of angular particles in which the particles can break.

Also, comparisons between simulations results and observations obtained from experimental tests show that the method presented for modeling breakage, can help us to have a qualitative view about the effect of breakage phenomenon on behavior of granular materials.

References