Effect of Confining Pressure on Particle Breakage of Assemblies of Simulated Angular particles Using Discrete Element Method

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ABSTRACT: A model is presented to simulate the breakage of two-dimensional polygon-shaped particles with using DEM (Discrete Element Method). In this model, initial shapes of the particles can be defined and each intact particle is then replaced with smaller inter-connected sub-particles with bonds sticked them together. If the bond breaks, breakage will happen. In this paper, influence of particle breakage in an assembly is investigated. The results are presented in terms of macro and micro mechanical behavior for different confining pressures. To do so, two series of biaxial test simulations (breakage enabled and disabled) are preformed and the results are compared.

1 Introduction

Soil structures such as breakwaters and rockfill dams are consisted of non-continuous media whose stability is due to the behavior of their granular body. There are lots of different factors influencing on shear resistance and behavior of granular materials such as mineralogical composition, particle grading, size and shape of particles, fragmentations of particles and stress conditions. Breakage of particles will happen in such high structures especially in the lower layers where there are levels of significant pressure caused by the upper layers. Crushing of large particles into smaller ones result in changes in the design gradation curve; therefore, the mechanical behavior of granular material alters.

In this paper, behavior of soil media which is discrete, is studied. Introducing a new method for simulating particle breakage, and by using Discrete Element Method, some simulated tests are performed under different levels of pressures and the assembly behavior in terms of macroscopic and microscopic parameters is discussed.

2 Review

Influence of confining pressure and consequently particle breakage on shear strength and deformability of granular materials can be studied using experimental tests such as Triaxial and unconfined compression tests((Marsal 1967, Bertacchi et al. 1970, Fumagali et al. 1970, and Marachi et al. 1972, Ansari & Chandra 1986, Venkatachalam 1993,Varadarajan et al. 2003). Marshal (1967) found out that the most important factor affecting both shear strength and compressibility is the phenomenon of fragmentation. Varadarajan et al. (2003) have investigated the behavior of two dam site rock materials (Ranjit Sagar and Purulia) in triaxial compression tests which the former was rounded and the latter angular particles. Particle breakage was observed during shearing. It is found out that breakage is affected by the particle size and confining pressure and it increases with both factors. Also it is noted that angular particles are more susceptible to be broken than rounded particles. In both groups, the confining pressure has the same effect on the behavior; the mobilized shear strength reduces but the compressibility grows up. In addition to experimental tests that require large size of specimen, there are numerical methods with computer technology to simulate discontinuous media to model breakage of brittle bodies with the help of

Discrete Element Method (DEM). Of these, are the approach used by Cundall (1978), the method based on simultaneous utilization of Molecular Dynamics (MD) (Kun et al. 1996) and the 3D approach used by Robertson & Bolton (2001) and McDowell & Harireche (2002). In this research, a different method is employed as described in the following.

3 Particle Breakage Simulation

In this research, simulation of biaxial test is performed on assemblies of 500 particles within 1500 sub particles using personal computer (PC). For this purpose, the program POLY (Mirghasemi et al. 1997) which is a modified version of DISC(Bathurst, 1985), to simulate two-dimensional polygon-shaped particles, is developed to model assemblies of irregularly shaped particles with the ability of breakage.(Mousavi Nik, 2000 and Seyedi Hosseininia, 2004).

In this method it is assumed that each intact particle P consists of smaller bonded particles like P_1, P_2, …and P_n. Particle P is called the Base Particle and the particles P_1 to P_n are called Sub-Particles. In other words, a particle can be divided into pieces during the test. Therefore shape of the particles obtained from breakage of the primitive particle is specified from the beginning. The sub-particles are rigid bodies and are not breakable or deformable. The base particles are not deformable but breakable. The both base and sub-particles are arbitrarily convex polygon shaped. It is assumed that each of the two adjacent sub-particles is connected with a connection at the middle of their common edge (Points m_1 and m_2 in Figure 1). This connection plays the role of the bond between two bonded sub-particles. If the stress formed in the connection exceeds its final bearing capacity during the simulation, two connected sub-particles are separated and breakage takes place.

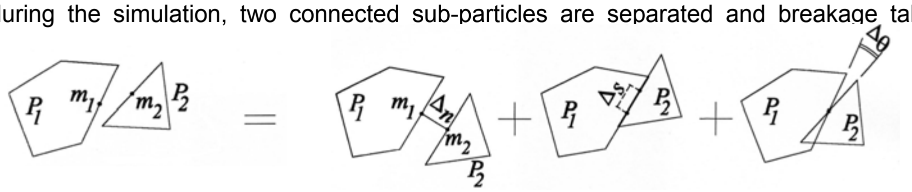


Figure 1. Replacement of the relative displacement of two sub-particles with three normal, shear and rotational components

For modeling the connection between two bonded particles, two transitional and one rotational springs are introduced. One of the transitional springs that are perpendicular to the common face of particles is called the normal spring and the other one which is parallel to the common face is the shear spring. Moment and forces at the connection are transferred through rotational and transitional springs, respectively. They can be calculated according to relative displacement of two sub-particles at each simulation cycle.

Figure 1 shows a base particle P in an assembly of particles. Due to interaction between particles, the forces and moments are induced at base particle's contact points. Movements of sub-particles P_1 and P_2 relative to each other caused that points m_1 and m_2 are no longer coincident with each other. To determine the force and moment applied on each sub-particle, the relative displacement of the two sub-particles is replaced with its three components, Δ_n (normal displacement), Δ_s (shear displacement) and Δ_θ (rotational displacement). Hence, the normal and shear forces and the moment at the contact point can be expressed as follows:

$$F_{n\text{-}Bond} = K_{n\text{-}Bond} \cdot \Delta_n$$
$$F_{s\text{-}Bond} = K_{s\text{-}Bond} \cdot \Delta_s \qquad [1]$$
$$M_{Bond} = K_{\theta-Bond} \cdot \Delta_\theta$$

where $K_{\theta-Bond}$ is the stiffness of the rotational spring and K_{nBond} and $K_{s\text{-}Bond}$ are unit length stiffness of the normal and shear springs, respectively. Values of these parameters are considered to be proportional to the stiffness of the particles.

If either shear, compressive or tensile stress at the bond between the two adjacent sub-particles exceeds its final admissible value, the bond is broken and particle breakage will happen.

The bond bearing capacity obeys from the Coulomb failure criterion for rocks which is extended in both compressive and tensile stresses but they are limited by magnitudes of stresses obtained from unconfined compressive strength and Brazilian tensile strength tests respectively (Seyedi Hosseininia, 2004). The same slip model acts between unbonded particles in contact, but no limit in the upper bound for compressive strength exists. Also no tensile stress can be tolerated between unbounded particles. The greater the normal stress on the slip surface, the stronger is the shear resistance. If the shear force between objects in contact exceeds the resistance, slip occurs.

4 Simulations

Simulation of two series of biaxial compression tests is fulfilled with four levels of confining pressure of 0.5, 1.0, 2.0 and 4.0 MPa to investigate the particle breakage in a granular media. In the series test A particles are rigid with no ability in fragmentation while in the series test B the rigid particles are breakable. The area which the particles are held in is a circle.

Table 1. Parameters used in test A (Breakage disabled)

Normal and tangential stiffness (N/m)	2.0×10^7
Unit weight of particles(kg/m^3)	2500
Transitional damping coefficient(1/sec)	75
Rotational damping coefficient(1/sec)	450
Time step(sec)	3.2e-4
Strain rate	0.005

Table 2. Parameters used in test B (Breakage enabled)

Normal and tangential stiffness (N/m)		2.0×10^7
Unit weight of particles (kN/m^3)		2500
Transitional damping coefficient (1/sec)		150
Rotational damping coefficient (1/sec)		900
Time step (sec)		1.52E-4
Strain rate		0.005
Module of elasticity (E) (MN/m^2)		9.0×10^4
Rock Strength Parameters	Compressive Strength (MN/m^2)	350
	Tensile Strength (MN/m^2)	35
	Intercept (MN/m^2)	75
	Coefficient of Static Friction	1.60

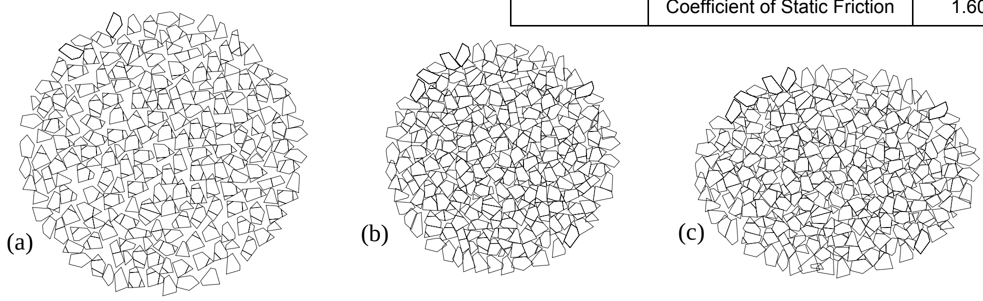


Fig.2. Simulation stages: (a) Initial generated assembly of particles, (b) Isotropically compacted assembly, (c) Sheared assembly at last stage of biaxial test (after Seyedi Hosseininia, 2004).

The parameters used for tests A and B are summarized in Table 1 and 2, respectively. Each test includes three stages. At first, the initial computer-generated assembly of particles (Fig.2 (a)) was compacted, then subjected to a confining pressure defined by the user and finally the assembly was sheared biaxially at a constant deviotoric strain rate. The inter-particle friction coefficient is set to 0.5 for all tests and the particles are assumed to be cohesionless at the contact. Also the particles have no weight.

5 RESULTS

5.1 Macromechanical behavior

The macroscopic results of biaxial simulation tests in both series A and B are presented in the form of curves of sin ϕ mobilized (Fig.3) and volumetric strain (Fig.4) versus axial strain.

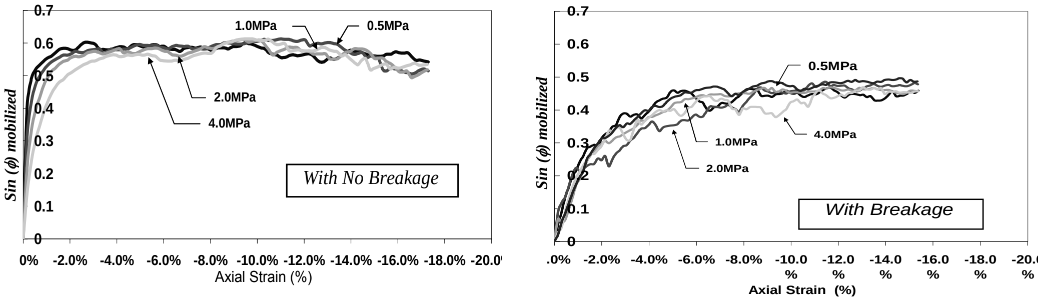


Fig.3. Relationship between sin $\phi_{\text{mobilized}}$ and axial strain

As shown in figure 3, the shear strength (sin ϕ mobilized), which can be defined by equation 2 increases smartly and then reaches to a constant value in tests A, but in the other series, it grows up gradually.

$$Sin\phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \frac{\sigma_1/\sigma_3 - 1}{\sigma_1/\sigma_3 + 1} \quad [2]$$

The effect of confining pressure on shear strength is in reverse where the higher pressure results in the lower mobilized friction angle in both tests. Also the axial strain at failure increases with increasing confining pressure.

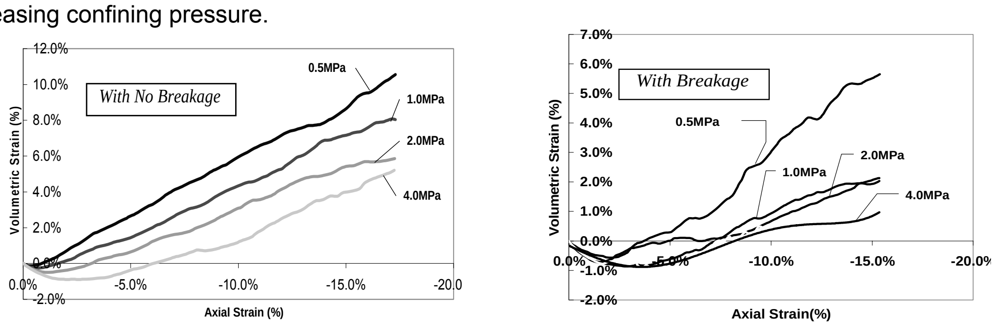


Fig.4. Volumetric strain versus axial strain during the test

In Figure 4, it can be seen that assembly with no breakage has a more dilative behavior than that with the ability of fragmentation. The higher confining pressure on the specimen causes to compress it more and does not let the sample dilate. On the other hand, under higher pressures particles have more tendencies to be broken. This causes the assembly to show a more compressive behavior under larger confining pressures. This trend can describe the reason for the reduction of sin $\phi_{\text{mobilized}}$. On overall, for both series of simulations, the more the assembly dilates, the larger is the shear resistance. The same result has been obtained in experimental test results (Marshal, 1967, Furnagalli et al., 1970, Varadarajan et al., 2003).

Table 3 shows obtained frictional angle of the assembly in different confining pressures. Also Figure 5 illustrates the variation of particle breakage degree (in percent) which has been tracked during different biaxial shear tests. This diagram confirms that higher degree of breakage is achieved when the larger value of confining pressure is used in the simulations. Marsal (1973) showed that at the beginning of the test, larger particles that contain more flaws and defects, break and it is why the breakage rate is high at the beginning.

Table 3. Comparison of internal friction angle in simulated tests

Confining Pressure	***With no Breakage***	***With Breakage***
0.5 MPa	37.6°	35.0°
1.0 MPa	36.2°	27.4°
2.0 MPa	34.8°	26.7°
4.0 MPa	33.4°	26.1°

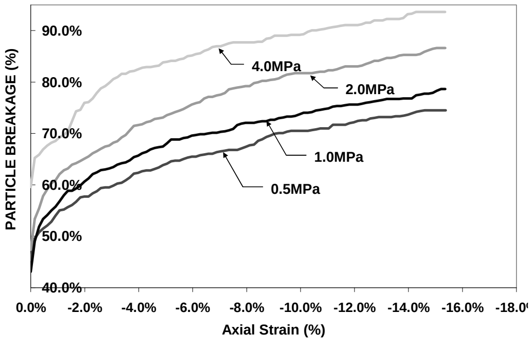


Fig.5. variation of particle breakage with axial strain

5.2 Microscopic Behavior

It is possible to find the distribution of contacts in a granular assembly in which the particles are carrying forces. For any angle θ, the portion of the total number of contacts in the system that are oriented at angle θ is $E(\theta)$. The distribution of contact normal orientations is described by a function such that the fraction of all assembly contact normals falls within the orientation interval $\Delta\theta$. Rothenburg (1980) showed that the distribution of such contacts takes the form

$$E(\theta) = \frac{1}{2\pi}\left[1 + a\cos 2(\theta - \theta_0)\right] \qquad [3]$$

where a is referred to as the parameter of anisotropy, and θ_0 is the major principal direction of anisotropy. The parameter a is proportional to the difference in the number of contacts oriented along the direction of anisotropy and in perpendicular direction.

The magnitudes of the contact forces in an assembly with irregular geometry vary from contact to contact. The average contact force acting at contacts with an orientation can be decomposed into an average normal force component, $\bar{f}_n^{\,c}(\theta)$, and an average tangential force component, $\bar{f}_t^{\,c}(\theta)$. By averaging the contact forces of the contacts falling within the group of similar orientation and following the same logic as for the contact normals, symmetrical second-order tensors may be introduced to describe average normal contact forces and average tangential contact forces. The average normal and tangential contact force tensors can be defined as (Bathurst, 1985)

$$\bar{f}_n(\theta) = \bar{f}_n^{\,0}[1 + a_n \cos 2(\theta - \theta_0)] \qquad [4]$$

$$\bar{f}_t(\theta) = -\bar{f}_n^{\,0}[a_t \sin 2(\theta - \theta_0) \qquad [5]$$

where a_n and a_t are the coefficients of normal and tangential force anisotropy respectively. θ_0 is the major principal direction of force anisotropy and $\bar{f}_n^{\,0}(\theta)$ is the average normal contact force from all assembly contacts. The general expression for the average stress tensor can now be written as

$$\sigma_{ij} = m_v l_0 \int_0^{2\pi} \left[\bar{f}_n^{\,c}(\theta) n_i^c n_j^c + \bar{f}_t^{\,c}(\theta) t_i^c n_j^c\right] E(\theta)\, d\theta \qquad [6]$$

where m_v is the average number of contacts per unit area (volume), l_0 is the assembly average contact vector length (average distance from the particle centroid to the contact point), $n_i^{\,c}$ is the contact normal vector, and $t_i^{\,c}$ is the contact tangent vector. Rothenburg & Bathurst (1989, 1992) derived a relationship between the measure of shear stress and the microscopic parameters a, a_n, a_t according to equations (2)-(5). For the case when the directions of anisotropy in contact forces and contact orientations coincide, as in a biaxial test, the relationship is as follows:

$$\left(\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}\right) = \left(\frac{a + a_n + a_t}{2 + a_n \times a_t}\right) \qquad [7]$$

The simplified expression suggests that the capacity of a cohesionless granular assembly is directly attributable to its ability to develop anisotropy in contact orientations or to withstand directional variations of average contact forces. Equation (7) was evaluated for the media with disc-shaped particles, elliptical particles, angular particles (Mirghasemi et al., 1997) and for the media in which the particles have the ability of breakage (Seyedi Hosseininia, 2004).

One way of investigating how a microstructure of granular assembly evolves during the shearing process is to study the change in the number of contacts in the assembly or the average coordination number of the system.

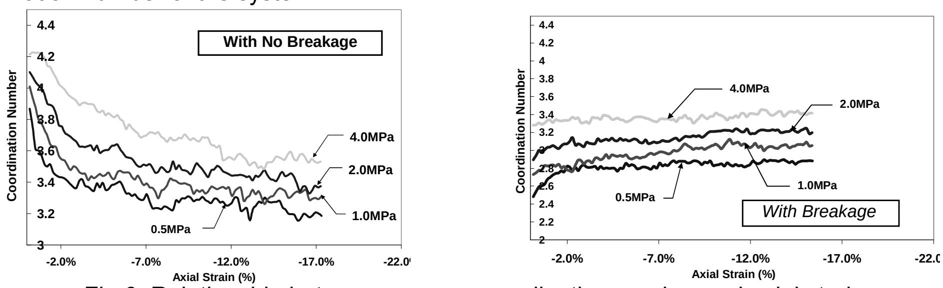


Fig.6. Relationship between average coordination number and axial strain

Fig. 6 presents the evolution of the average coordination number during shear deformation. At the beginning of each test, some contacts were created owing to the elastic compression from hydrostatic stress. The coordination number in tests B (with breakage) is less than that of the tests A (with no breakage). Also the trends are different from each other during shearing process. In test series A, it decreases rapidly with axial strain and comes to be constant, while in test B, it grows gradually towards a constant value in high confining pressures. During each test in both series of tests, contacts in the assembly began to degrade as the axial stress increased, mainly in the horizontal direction. But in test series B, particle breakage is also happening at the same time and this phenomenon results more development of contacts between particles. The effect of increasing the confining pressure on coordination number can be observed from the figures. The higher confining pressure, the more contacts are induced in the assembly.

The variation of the contact normal anisotropy (parameter a) as a function of axial strain is illustrated in Fig. 7. It describes the degree of anisotropy in contact orientations. The coefficient of fabric anisotropy evolves to the maximum values as contacts are lost, mostly oriented along the direction of tensile strain (horizontal direction). But this growth is more rapid in tests A than in test B where particles can break. The Confining pressure has a reverse effect on this coefficient, but this rule is not always correct in the test series B.

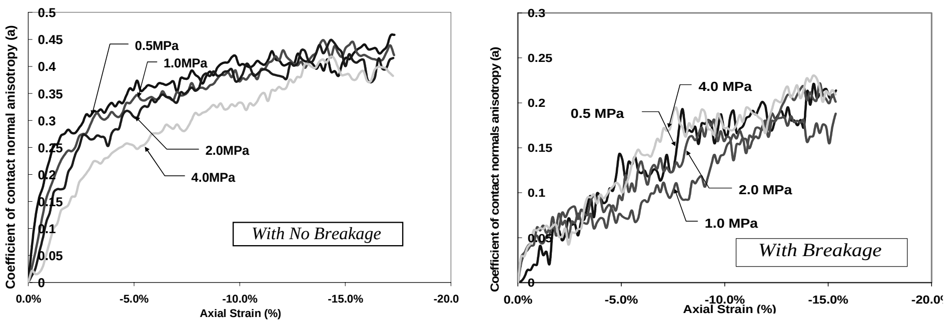
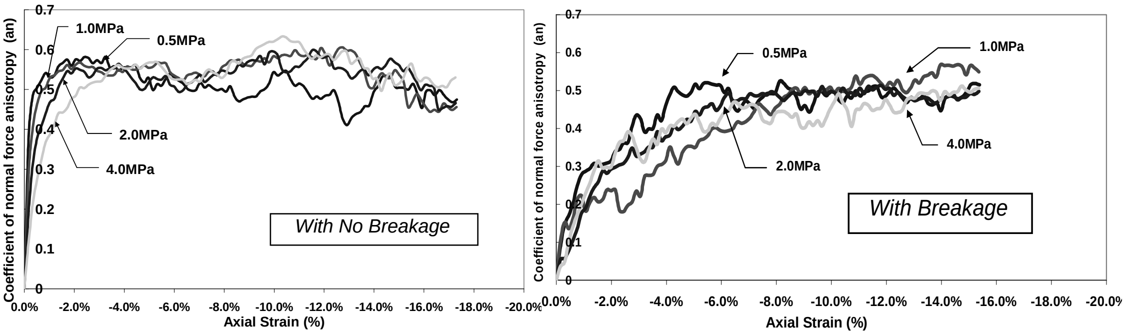
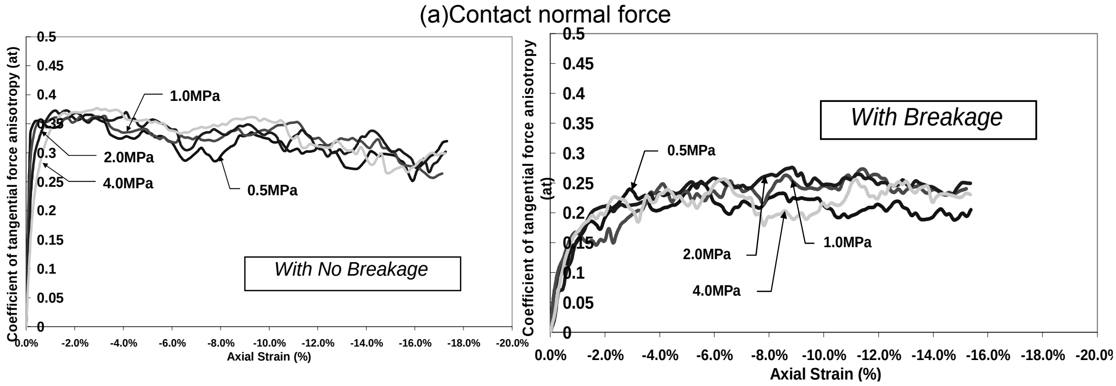


Fig.7. Evolution of contact normal anisotropy coefficient

Figure 8 presents the development of anisotropy in normal and tangential contact forces by the variation of coefficients a_n, a_t with axial strain during the simulations. In test series A, as the axial strain increases, a_n, a_t show a rapid growth at lower axial strain, followed by a reduction after the maximum value. This is because of loss of contacts and also the loss of the capacity of chains of particles to sustain high forces both for normal and frictional resistance. In contrast to test A, when the particles can break, the force anisotropy parameters show a gradual increase which reaches at a constant value. This behavior is justified while the particles cannot tolerate the imposed forces and breakage happens, therefore, particles can not make a chain to show a peak. Also, it can be observed that assemblies with lower confining pressure can provide more anisotropy in the media with subsequent higher shear strength.



(a)Contact normal force



(b) Contact tangential force

Fig.8. Evolution of force anisotropy coefficient

6 Conclusion

The results of two simulated series of biaxial tests with several confining pressures indicated that high confining pressure leads to decrease shear strength and increase granular material compressibility of the media. In return, the dilatancy falls down. Also the rate of particle breakage was investigated. The higher the confining pressure, the more the degree of breakage is. The results are similar to data obtained from experimental tests on real rockfill materials.

The influence of confining pressure on the variation of micromechanical parameters was studied. The assembly in which breakage is enabled, the coordination number remains almost constant during the test, but in the other group, it decreases along with axial strain. As observed, the magnitude of normal contact, normal force and tangential force anisotropy coefficients are smaller in the case of breakable particles than those in rigid particles. But the confining pressure has a reverse effect on the anisotropy coefficients. The shear strength of granular assembly is directly attributable to the ability to develop anisotropy. Also, Comparisons between simulations results and observations obtained from experimental tests shows that the method presented for modeling breakage, can help us to have a qualitative view about the effect of breakage phenomenon on behavior of granular materials.

7 References

Cheng, Y.P., Nakata, Y. & Bolton, M.D. (2003), Discrete element simulation of crushable soil, Geotechnique 53, No. 7, 633-641.

Cundall, P.A. (1978)."Ball – A Computer Program to Model Granular Media Using Distinct Element Method", Technical Note TN-LN-13., Advanced Technology Group, Dames and Moore, London.

Kun, F. and Herrmann, H.J. (1996). "A Study of Fragmentation Process Using A Discrete Element Method." Comput.Methods Appl. Mech. Engrg. 138, pp.3-18.

Marsal, R.J. (1967). "Large Scale Testing of Rockfill Materials", J.of the Soil Mechanics and Foundation Division, Vol.93, No. SM2.

Mirghasemi, A.A., Mousavi Nik, R. (2001), "Influence of particle breakage on mechanical behavior of assemblies of two-dimensional polygon-shaped particles", Proc. of 54th Canadian Geotechnical Conference, Calgary, Alberta, 614-620

Mirghasemi, A.A., Rothenburg, L. and Matyas, E.L.(1997) Numerical Simulations of Assemblies of Two-Dimensional Polygon-Shaped Particles and Effects of Confining Pressure on Shear Strength, Soils and Foundations, Japanese Geotechnical Society,Vol.37,No.3,pp.43-52.

Mirghasemi, A.A., Rothenburg, L. and Matyas, E.L.(2002) Influence of particle shape on engineering properties of assemblies of two-dimensional polygon-shaped particles, Geotechnique 52, No.3, 209-217

Mirghasemi, A.A., Seyedi Hosseininia, E. (2004), "Micromechanical considerations of particle breakage using discrete element method", Proc. of 57th Canadian Geotechnical Conference, Quebec city, Quebec

Rothenburg, L., Bathurst, R..J. & Dusseault, M.B. (1989). "Micromechanical ideas in constitutive modeling of granular materials", Powders and Grains, Biarez & Gourves(eds), 1989 Balkema, Rotterdam. ISBN 906191 984 3

Seyedi Hosseininia, E., "Micromechanical Considerations of Particle Breakage using Discrete Element Method", M.Sc Thesis, Tehran University, Tehran, Iran

Varadarajan, A, Sharma, K.G., Venkatachalam, K. & Gupta, A.K., "Testing and modeling two rockfill materials", Journal of geotechnical and geoenvironmental engineering, ASCE, March 2003