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INVESTIGATION OF BEHAVIOR OF PARTICULATE MEDIA USING DISCRETE ELEMENT METHOD

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ABSTRACT

Many attempts have been made to find various relationships for different parameters and some kinds of constitutive models studying behavior of particulate media. All these models are based on concepts of continuous media which some of them have considered elastic behavior of soil too. Using a numerical method such as Discrete Element Method, one can figure out what is happening through a discontinuous media where soil particles have the main rules in introducing the shear strength and deformation characteristics of the media. The behavior of the media is more important when the particles have the ability of fragmentation. But most of the models presented before, have not consider this phenomenon. In this paper, the hyperbolic elastic model is investigated for an assembly of polygon shaped particles in two different test series. Also evolution of different macro parameters of the assembly such as volume strain, angle of friction, angle of dilatancy and elastic modulus are studied during the tests both for non-breakable and breakable soil particles.

RÉSUMÉ

Beaucoup de tentatives ont été faites de trouver de divers rapports pour différents paramètres et quelques genres de modèles constitutifs étudiant le comportement des médias particulaires. Tous ces modèles sont basés sur des concepts des médias continus que certains d'eux ont considérés le comportement élastique du sol aussi. En utilisant une méthode numérique telle que la méthode discrète d'élément, on peut figurer hors de ce qui se produit par des médias discontinus où les particules de sol ont les règles principales en présentant les caractéristiques de résistance au cisaillement et de déformation des médias. Le comportement des médias est plus important quand les particules ont les capacités de la fragmentation. Mais la plupart des modèles ont présenté avant, démuni considèrent ce phénomène. En cet article, le modèle élastique hyperbolique est étudié pour un ensemble des particules formées par polygone de deux séries différentes d'essai. En outre l'évolution de différents macro paramètres de l'ensemble tels que la contrainte de volume, l'angle du frottement, l'angle de l'épaississement et le module élastique sont étudiés pendant les essais pour les particules non-cassables et cassables de sol.

1. INTRODUCTION

The crushability of soil particles has been studied experimentally by a number of researchers and Influence of particle breakage on internal friction angle and deformability of granular materials have been investigated with tests such as triaxial and unconfined compression tests (Marsal 1967, Bertacchi et al. 1970, Fumagali et al. 1970, and Marachi et al. 1972, Venkatachalam 1993, Varadarajan et al. 2003).

Triaxial tests of sands show a strong dependence of strength and dilatancy behavior on both relative density and stress level, relating to grain crushability (Bolton, 1986). An empirical characterization of peak angle of internal friction of sands in relation to those two parameters was reasonably successful, although inherent anisotropy due to bedding was also seen to be important.

In recent years, along with the progress of numerical methods and computer technology, different methods have been used to model breakage of brittle bodies with the help of Discrete Element Method (DEM). Discrete element simulation of perfectly elastic and infinitely strong grains provides many insights into the deformation of granular media (Thornton, 2000). Among these methods, are the method based on simultaneous utilization of Molecular Dynamics (MD) (Kun et al. 1996) and the 3D approach used by Robertson & Bolton (2001) and McDowell & Harireche (2002).

Breakage of particles is very important whereas it influences on the stability and the deformation of an earthwork structure where there is a high pressure on the layers. This is happening in the underlying layers, bearing significant weight of the upper layers. The soil grains in the underlying layers are subjected to significant stress magnitudes. The induced high stresses may cause the particles to be broken. Particle breakage and crushing of large particles to smaller ones, results in changes in grain size (gradation) curve;

therefore the mechanical behavior of granular material alters. In general, shear resistance and behavior of granular materials depends on different factors such as mineralogical composition, particle grading, size and shape of particles, fragmentations of particles and stress conditions. Here, we only emphasize on the particle crushing.

In this paper the results of biaxial test witch are simulated with DEM are presented. These tests are performed in two series, one with unbreakable particles and the latter with particles which can be broken through both confining stage and shearing process. For these tow series, the strength and deformation parameters are compared with together.

2. BREIF REVIEW OF MODELING

The program POLY (Mirghasemi et al. 1997) which is a modified version of DISC (Bathurst, 1985), to simulate two-dimensional polygon-shaped particles, is developed to model assemblies of irregularly shaped particles with the ability of breakage (Mousavi Nik, 2000 and Seyedi Hosseininia, 2004). It has been tried to model the particle breakage in a way that less number of particles and computational effort are needed (Mousavi Nik, 2000).

In the present research, simulation of biaxial test is performed on assemblies of 500 particles within 1500 sub-particles using personal computer (PC).

In this method it is assumed that each soil particle can break through pre-defined straight lines with certain direction and position. The lines are determined in a way that two commonly observed behavior can be simulated. These two kinds of behavior are cracking of particle vertexes and cracking across a particle that divides particle into pieces. For example, according to Figure 1(a), it is assumed that the particle P can only break through the lines d_1, d_2 and d_3; therefore shape of the particles obtained from breakage of the primitive particle is specified from the beginning. Thus in this method, each uncracked particle like P consists of smaller bonded particles like P_1, P_2, …and P_n. Particle P is called the Base Particle and the particles P_1 to P_n are called Sub-Particles. The sub-particles are considered to be rigid bodies. They are not breakable and not deformable. The base particles are not deformable but they are breakable. The both base and sub-particles are arbitrarily convex polygon shaped. The detailed method for simulating the bond between these sub-particles is presented in references [15] and [16].

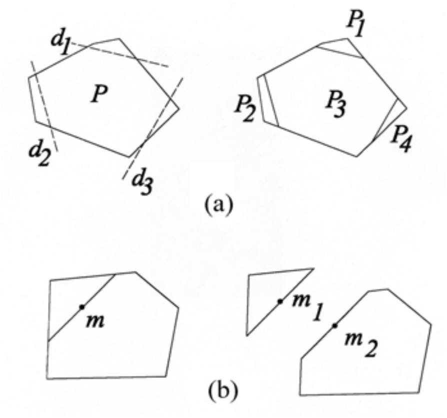


Figure 1. Breakage modeling
(a) Base particle P and its sub-particles.
(b) The bond points of two adjacent sub-particles

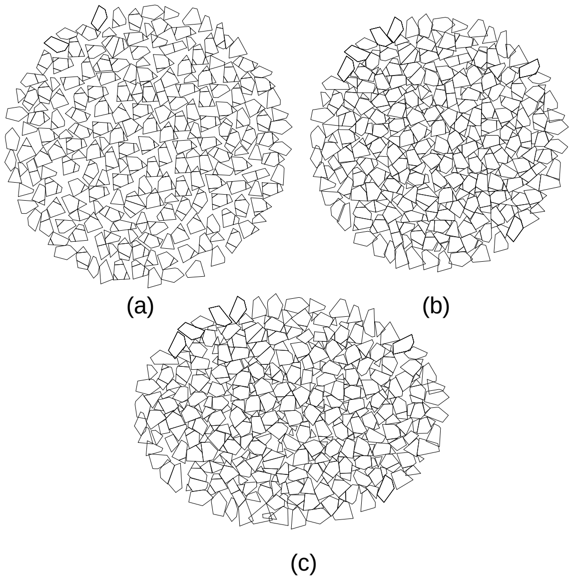
At the present research, an investigation is made to study the influence of particle breakage on behavior of granular media.

3. TEST SIMULATIONS

Simulation of two series of biaxial compression tests is fulfilled with five levels of confining pressure of 0.5, 1.0, 2.0, 4.0 and 8.0 MPa to investigate the particle breakage in a granular media. In the series test A particles are rigid with no ability in fragmentation while in the test series B the rigid particles are breakable. The area which the particles are held in is a circle.

Each test includes three stages. At first, the initial computer-generated assembly of particles (Figure 2(a)) was compacted, then subjected to a pre-defined confining pressure and finally the assembly was sheared biaxially at a constant deviatoric strain rate of 0.005. It is needed to let the assembly to become stable after each simulation stage, i.e. the particles do not move while there is no unbalanced force on them.

The inter-particle friction coefficient is set to 0.5 for all tests (ϕ_μ=26.6^o) and the particles are assumed to be cohesionless at the contact. Also the particles have no weight. Three stages of simulation for a breakable test are shown in Figure 3. Both tests are fulfilled to the axial strain level of 16%.


(a) (b)

(c)

Figure 2. Three simulation stages for breakable particles
(a) Initial generated assembly of particles,
(b) Isotrpically compacted assembly,
(c) Sheared assembly at last stage of biaxial test (after Seyedi Hosseininia, 2004).

4. TEST RESULTS

The results of biaxial test simulations in confining pressure level of 2.0MPa ($P_{c.p.}$=2.0MPa) are presented in the form of curves of stress obliquity and volumetric strain versus axial strain (Figure 3).

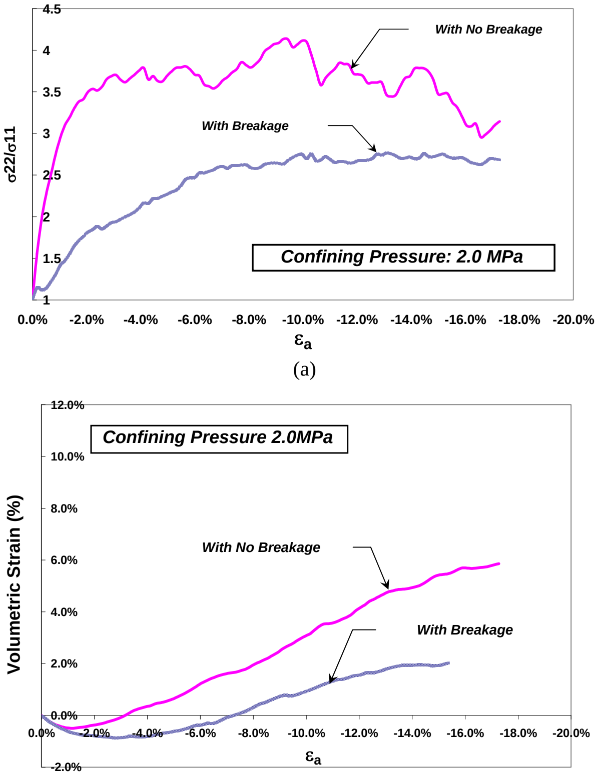
As shown in Figure 3(a), the shear strength (stress obliquity) in test series A increases rapidly at the initial strain, then it reaches to a peak value gradually and finally it decreases and reaches to a constant value.

In test B, the trend is different where the stress ratio is increasing gradually till it reaches to a constant value. It seems that particle breakage has a decreasing effect on shearing resistance of the assembly.

In both tests, the shear strength is mobilizing along the axial strain and the difference in their behavior is due to the occurrence of interlocking. That is why we can observe some peak values in test A and no more in test B. Particles in test A that can not be breakable, are interlocked within each other and this happening can cause a peak strength in the behavior following a reduction when this interlocking is collapsed and particles trend to slide on each other with their inter particle resistance angle (ϕ_μ).

Hence the shear resistance of soil mass is made up of two components: (a) one whose magnitude is controlled by $\phi\mu$; and (b) a second whose magnitude is related to the degree of interlocking. The greater the degree of interlocking, the higher the overall shear resistance is. Finally, as shear motion continues, the interlocking decreases and consequently, the shear force necessary to continue the motion must also decrease (in large strains).

Despite in test A, the peak shear strength is not seen in test B since particles can not bear the implied stresses and no interlocking can happen, but the broken particles are trying to be show the largest power of strength at the large strains where the deformation is so large that the structure may be damaged already.



(a)

(b)

Figure 3. Relationship between stress obliquity and volumetric strain versus axial strain($P_{c.p.}$=2.0MPa)

In general, the angular particles have dilative behavior (Sharma, 1967; Varadarajan et al., 2003). It is a remarkable fact that a dense assembly of soil particles, when compressed in one direction, actually increases in volume. The fact was first observed by Osbourne Reynolds in 1885. Figure 3(b) shows the volumetric strain of both test series during biaxial test. By comparing the value of volumetric strain in both groups, it seems that the assembly with no breakage has a more dilative behavior than that with the ability of fragmentation. In other words, the more the assembly dilates, the larger is the shear resistance. In test B, particles cannot undergo the forces imposed on them and breakage happens, therefore smaller particles fill the voids and let the other particles move freely. This causes the assembly to show a compressive behavior in larger axial strains followed by increasing volumetric strain. This trend can justify the reduction of stress ratio in test B. The same result has been obtained in experimental test results (Marshal, 1967, Fumagali et al., 1970).

Marshal(1967) by performing Triaxial compression tests on coarse granular materials found out that the most important factor affecting both shear strength and compressibility is the phenomenon of fragmentation undergone by a granular body when subjected to change in its state of stress both during uniform compression stage and during deviatoric load application. Also the results showed that in granular media, the compressibility is a consequence of complex phenomenon that takes place as a result of displacements between particles combined with the particle breakage. Varadarajan et al.(2003) have investigated the behavior of two dam site rock materials (Ranjit Sagar and Purulia) in triaxial compression tests which the former consisted of rounded and the latter angular particles. During the shearing stage of the triaxial test, compression, rearrangement and breakage of particles took place. The rounded material exhibited continuous volume compression, while the angular particles dilated and expanded after initial compression in volume. Granular materials provide a high degree of interlocking and cause dilation during shearing. Also they observed that a greater degree of particle breakage occurs with the larger particles because of the greater force per contact (Lame and Whitman 1969). The effect of increase in interlocking is to increase the shearing resistance, while the effect of breakage of particles is vice versa. Also it is noted that angular particles are more susceptible to break that rounded particles.

Figure 4 illustrates the variation of bond breakage degree (in percentage) which has been tracked during different biaxial shear tests. The breakage percentage shows the ratio of broken bonds to total number of bonds. This diagram confirms that higher degree of breakage is achieved when the larger value of confining pressure is used in the simulations. Having performed Triaxial tests on rockfill, Marsal (1973) showed that at the beginning of the test, larger particles that contain more flaws and defects, break and it is why the breakage rate at the beginning of the test is high. At the primitive stages of the test, the smaller particles, produced by larger particles breakage, are located in the voids between the other intact large particles and consequently have no role in transferring the force to their neighboring particles. After compaction of assembly during next stages, the gaps between particles become smaller and the small particles can play their role in transferring the force to the adjacent particles. Thus the mean contact stresses decrease owing to the increase of particles surrounding each grain; therefore, the breakage quantity will reduce afterwards. Considering the total number of breakage in Figure 4, the rate of particle breakage is high at the beginning of simulation and then it slows down. Therefore variation of breakage rate versus axial strain (and consequently axial stress) during the simulated biaxial test is in agreement with the trend observed by Marsal (1973).

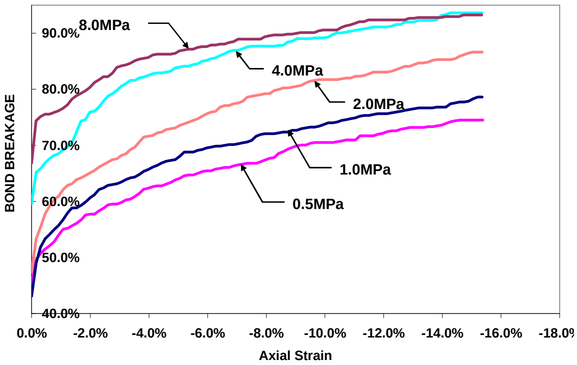


Figure 4. Degree of bond breakage in different confining pressure levels

In order to compare the obtained results from these simulations with the results from the experimental tests, the values of the maximum principal stress ratio $(\sigma_2/\sigma_1)_{max}$ in tests done by Gupta (2000), Venkatachalam (1993) and Marachi et. Al. (1969) with the simulation tests are shown in Figure 5 along the degree of breakage (B_g). The value of breakage is calculated from sieve analysis of rockfill sample as follows. Before testing, the sample is sieved using a set of standard sieves and the percentage of particles retained in each sieve is calculated. Due to the breakage of particles, the percentage of particles retained in large size sieves will decrease and the percentage of particles retained in small size sieves will increase. The sum of decreases in percentage retained will be equal to the sum of increases in percentage retained. The sum of decreases (or increases) is the value of the breakage factor (B_g) (Marshal, 1967). It is obvious that the simulation results are in good agreement with experimental tests which are close to the lower bound shown in figure 5.

One way of investigating how a microstructure of granular assembly evolves during the shearing process is to trace each particle displacements along the test. This is possible to see with numerical simulation in which the locations of all particles can be under controlled. Figure 6 represents the movement of all the particles in the unbroken test and all the sub-particle displacements in the test with breakable particles. In this sketch, the initial and final locations of all particles are connected to each other. It shows that all particles are trying to move towards the implied major and minor stresses during the test but the particles situated in the center of the assembly have the minimum movement during the test.

Investigating the evolution of the assembly from point of view of microscopic parameters is discussed in references [13], [14], [17].

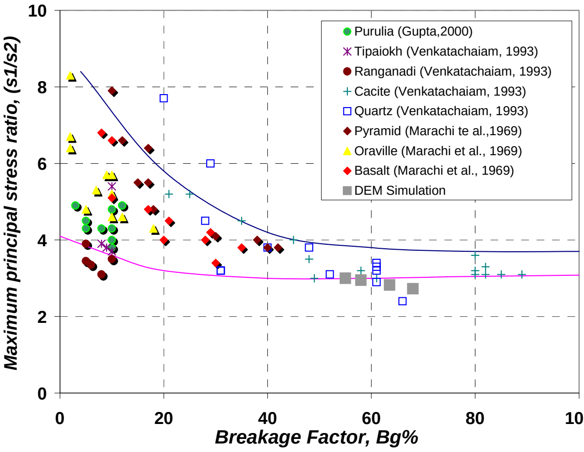
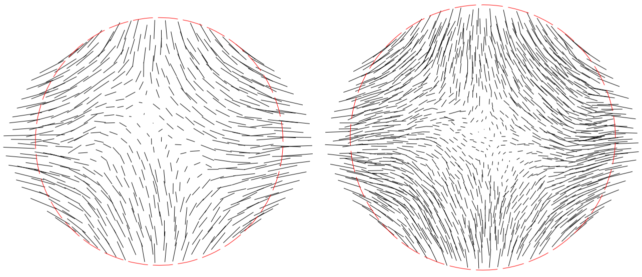


Figure 5. Variation of principal stress ratio versus degree of breakage (%) For results obtained from simulation biaxial test and experimental tests



(a) (b)

Figure 6. Displacement trajectories of all particles during the biaxial test in (a) Unbreakable test; (b) Breakable test

4.1. EFFECT OF CONFINING PRESSURE

In these tests, the effect of confining pressure can be observed, especially in series test A where the particles can not break. The bigger the confining pressure, the smaller the mobilized shear strength becomes. In test series B, this trend is some how felt but not for all range of applied strain during the test. Also the variation of volumetric strain along the axial strain along the shearing is presented (Figure 7). There are some important trends which should be noted:

1. As σ_v increases, the peak normalized tress decreases slightly. There is slight increase in the strain at which this peak occurs.
2. The normalized stress in the ultimate condition is more or less independent of σ_v.
3. The volume increase is less in the case of the tests with the larger confining stress.

These behavior is because of two factors including confining pressure and interlocking. Interlocking increases as the confining pressure increases; therefore the friction angle and dilatancy of assembly reduces.

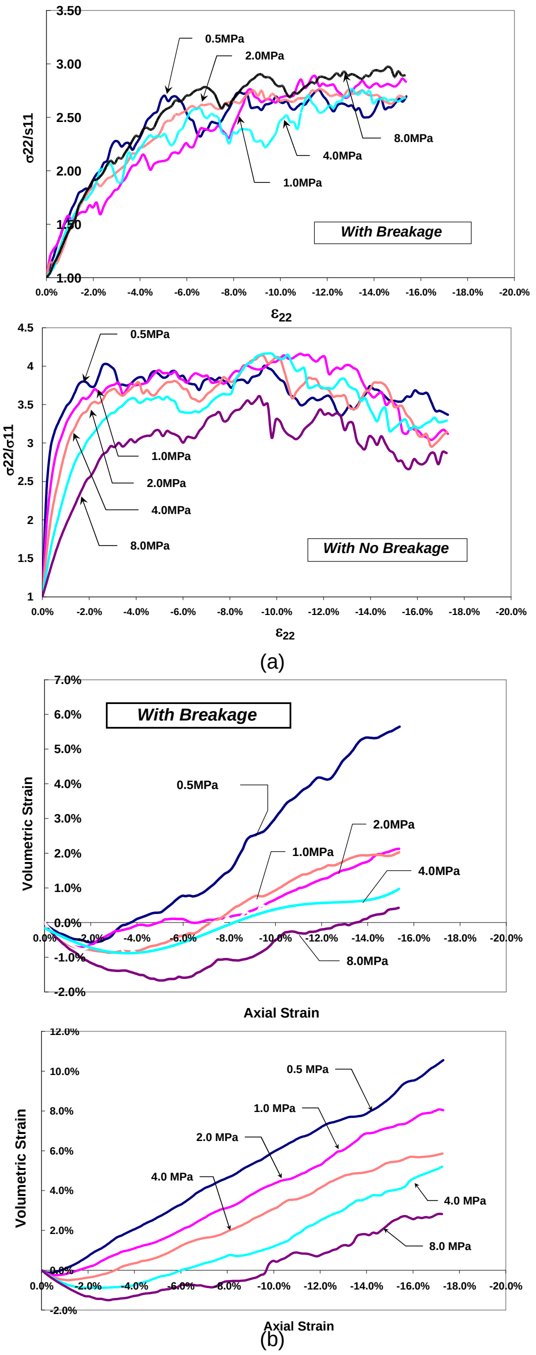


Figure 7. (a) Stress obliquity versus axial strain; (b) volumetric strain versus axial strain; for both groups of the tests

To figure out the effect of confining pressure on the shear strength, one can sketch the Mohr circles for the several tests to draw the Mohr-Coulomb Failure envelope by drawing a line tangential to all circles which should pass the origin point of the diagram, because the particles in assembly are cohesionless. In Figure 8, the Mohr envelopes for both tests are presented for 5 levels of confining pressure. The curvature in the envelope is generally true for granular soils tested using a wide range of confining stresses.

The way in which a straight line is fitted to a Mohr envelope will depend on what range of confining pressure ($\sigma_{c.p.}$) is of interest. In small range of confining stress, the straight line passes the origin with no cohesion, but in large ranges of stresses, the line can be drawn in such a way that it shows some cohesion intercept for the whole media. The confining stresses selected for the simulation of biaxial tests in this research, are selected high; therefore, it is expected to have some cohesion in the assembly, where the cohesion intercept in test A, shows the C'=230 kPa, but in the test series B with breakable particles, the fitted line passes the origin and cohesion intercept is zero.

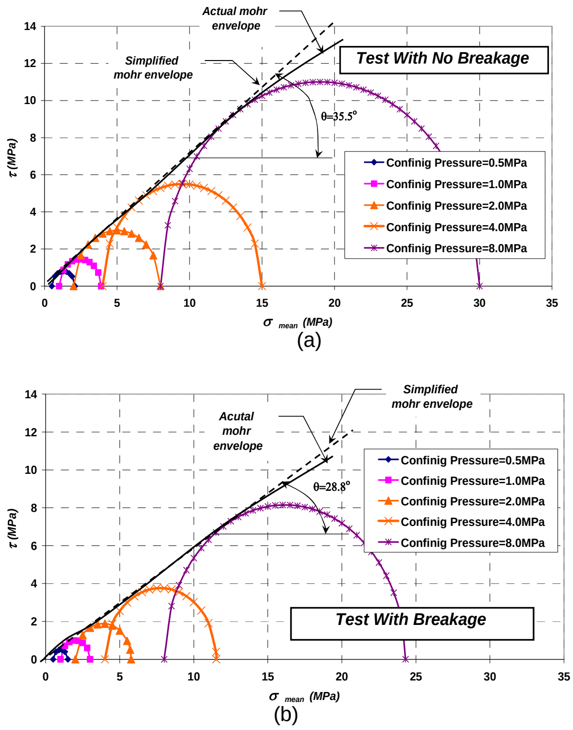


Figure 8. Mohr-Coulomb failure envelope derived from biaxial tests; (a) for test with no breakage; (b) for test with breakage

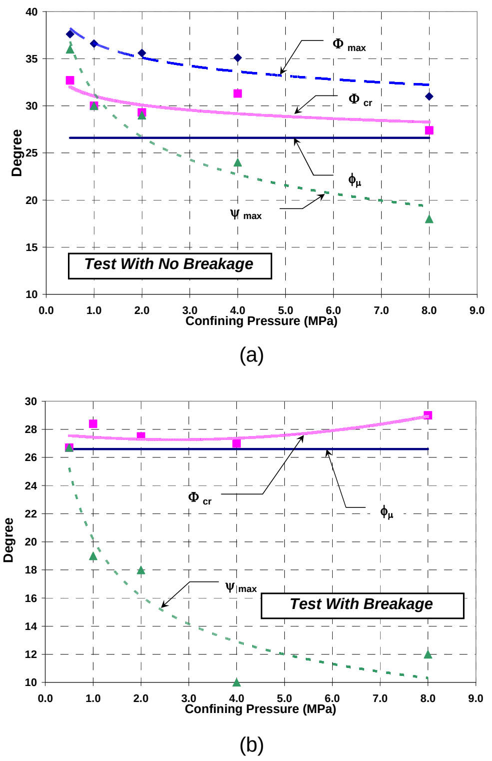
The variation of ϕ_{peak}, $\phi_{mobilized(=residual)}$ and ψ_{max} (peak dilation rate) along different confining pressure levels can be investigated in Figure 9. Frictional angle and peak dilation angle can be calculated from equations as described below:

$$\phi_{peak(or\ mobilized)} = Sin^{-1}(\frac{\sigma_2-\sigma_1}{\sigma_2+\sigma_1})_{peak(or\ mobilized)} \qquad [1]$$

$$\psi_{\max} = \tan^{-1}(\frac{d\varepsilon_v}{d\varepsilon_\gamma})_{\max} \qquad [2]$$

where:
σ_2: major principal stress
σ_1: minor principal stress
$d\varepsilon_v$: incremental volumetric strain ($=\varepsilon_2+\varepsilon_1$)
$d\varepsilon_\gamma$: incremental shear strain ($=\varepsilon_2-\varepsilon_1$)

In test A, non-breakable particles, the samples have reached to a maximum friction angle having a distance from the mineral to mineral inter particle friction angle (ϕ_μ). This is different in test series B, where the only mobilized friction angle of the assembly is getting closer to the ϕ_μ. In both tests the shear strength ($\phi_{mobilized}$) have a decreasing trend with increase of confining pressure except for the breakable assembly in a highest confining pressure of 8.0 MPa. Perhaps, it is because the most particles have been fragmented into several pieces (94%) in this high level of pressure causing alteration the particle size and the assembly consequently. So this new assembly shows a new behavior and shear strength. It can be concluded that high degree of particle breakage may produce a new product with higher resistance due to the alteration of the soil particles.



(a)

(b)

Figure 9. Variation of friction angle (ϕ_{max}, ϕ_{cr}), dilatancy angle (ψ), and inter particle friction angle (ϕ_μ) in different confining pressure levels in (a) test with no breakage; (b) test with breakage

Considering the dilation of the assembly, it is obvious that dilatancy angle (ψ) decreases with higher confining pressure especially where particle are breakable. In these two series of simulated tests, we have found that dilatancy angle is less that the inter particle angle (ϕ_μ), where in test B, this reduction is much more.

In this research the stress-strain behavior of the assemblies are compared with the hyperbolic elastic relationship suggested by Kondner and Zelasko (1963). They have stated that stress-strain curves of sand in standard triaxial compression can be fitted by a hyperbolic equation of the form:

$$\sigma_1-\sigma_3=\frac{\varepsilon_1}{a+b\varepsilon_1} \qquad [3]$$

where a and b are constants. a is reciprocal of initial tangential Young's modulus of the sample and b is reciprocal of the ultimate deviatoric stress.

As the confining pressure increases, the modulus increases. The modulus increases with the power of $\sigma_{c.p.}$ say, $E \propto \sigma^n$. n varies from 0.4 to 1.0. The larger values of the exponent tend to apply to loose soil samples. Janbu (1963) to relate the confining stress and modulus of Elasticity suggested:

$$E_i = K.P_a(\frac{\sigma_{c.p.}}{P_a})^n \qquad [4]$$

where:

E_i: initial Young's modulus
P_a: atmospheric pressure (0.1MPa)
K: loading modulus number
$\sigma_{c.p.}$: confining stress
N: exponent for behavior of loading

To investigate if this relationship is governing in these two simulated tests, all tests with confining pressure of 0.5, 1.0, 2.0, 4.0 and 8.0 MPa are considered. In logarithmic scale, five points of the aforementioned confining pressures are fitted by the best line. Figure 10 shows the best fitted lines for both tests A and B with their equation and R-squared values. As it is clear, in both groups, the points are placed in a good way that a straight line is fitted among them with R^2 of 0.99. This means that Young's modulus for various amounts of confining pressures can be estimated very well for this assembly.

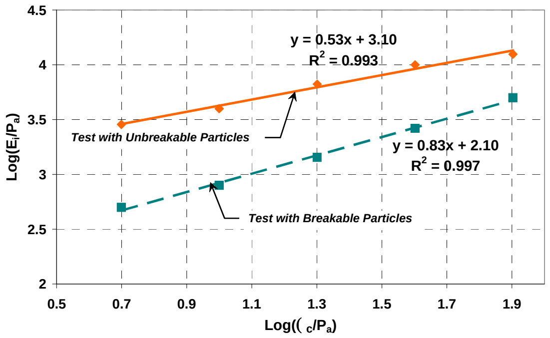


Figure 10. Determination of parameters K and n to estimate the initial Young's modulus with Jumbo formula (1963)

The other point is the value of *n* derived from these diagrams. The slope of the lines represents the value of *n*. As can bee seen, the *n* value in breakable test is 0.83 which is bigger than that of non-breakable test (0.53). This shows that the modulus of assembly with breakable particles grows slower than that in the assembly with particles which is not breakable. It is like to say that the breakable sample B treats such as a looser medium than sample A. It is worth noting that the intercept of this diagram denotes Log K. The final form of modulus relationships are as follows:

$$E_i = 12.7(\frac{\sigma_{c.p.}}{P_a})^{0.83} \qquad \text{For breakable assembly} \qquad [5]$$

$$E_i = 125.9(\frac{\sigma_{c.p.}}{P_a})^{0.53} \qquad \text{For non breakable assembly} \qquad [6]$$

Comparing equations [5] and [6], K factor in a breakable assembly (12.7) has been reduced to about 0.1 of that in a non breakable assembly (125.9). By dividing these two equations, we obtain:

$$\frac{E_{i\,breakable}}{E_{i\,non\,breakable}} = 0.2\sigma_{c.p.}{}^{0.3} \qquad [7]$$

Equation [7] shows the relation of deformation modulus of an assembly in two modes where the particles can break and can not break. This relationship is not a complete one because it does not include the degree of breakage of the particles.

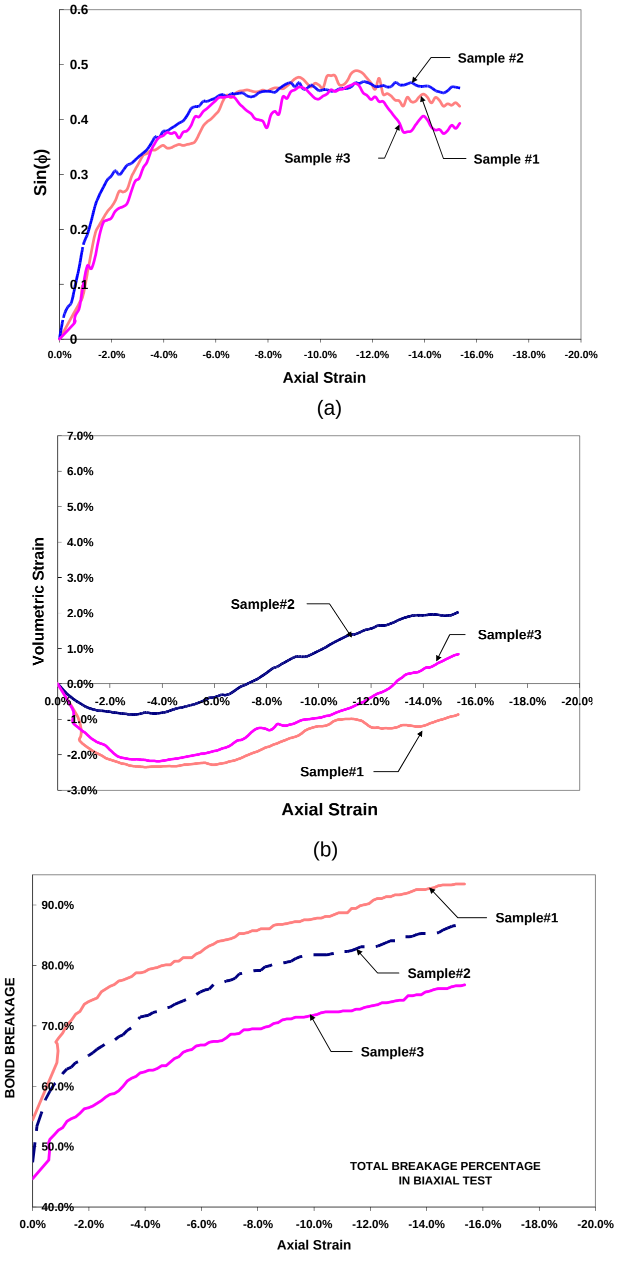
4.2. EFFECT OF ROCK STRENGTH ON THE BEHAVIOR OF THE ASSEMBLY

Three biaxial compression tests have been simulated with different rockfill materials to investigate the effect of strength on the behavior of the assembly. The strength parameters are shown in table 1. All these tests are performed in the same condition with confining pressure of 2.0 MPa. The inter particle frictional coefficient is held constant and equal to 0.5. Figure 11 shows the variation of $\text{Sin}\ \phi_{\text{mobilized}}$, volumetric strain and degree of total bond breakage (%) along the axial strain during the tests.

Table 1. Strength parameters for three rockfill materials in the tests with breakable particles

Rock Strength Parameters	Strength Parameters	Sample #1	Sample #2	Sample #3
	Compressive Strength (MN/m^2)	190	350	650
	Tensile Strength (MN/m^2)	19	35	65
	Intercept (MN/m^2)	40	75	140
	Coefficient of Static Friction *(tan(ϕ))*	1.6	1.6	1.6

As illustrated in Figure 11, although the amount of breakage is reduced with increasing rock strength, there is little sense of difference in their corresponding shear strength (Sin ϕ $_{\text{mobilized}}$). Perhaps this is because of high amount of breakage in all tests and also the degrees of breakage are almost at the same level (75%-92% at the end). The difference in behavior will be observed more clearly if the difference of breakage degree is in a larger range, since the assembly with highest rock strength (sample #3) shows a reduction in the mobilized Sin ϕ at the large axial strain, but it is almost equal to the other samples' at the beginning and the middle of the test.



(a)

(b)

(c)

Figure 11. Variation of (a) Sin ϕ $_{\text{mobilized}}$; (b) Volumetric strain; (c) Total bond breakage degree; with axial strain in confining pressure of 2.0 MPa

The other point which can be found out in this series of tests is the relation between the shear strength of the assembly and the dilation; the higher the assembly dilates, the higher the shear strength becomes. This point was also gained when the assembly was studied in different confining pressures.

5. CONCLUSIONS

The comparison of the two simulated series of biaxial tests indicated that breakage of the particles leads to decrease of internal angle of friction and increase of granular material compressibility. Also the rate of particle breakage in different confining pressures during biaxial test was investigated. The results are similar to data obtained from experimental tests on real rockfill materials.

The proposed hyperbolic elastic formula for sand was investigated for both groups of assemblies and it was observed that in both situations the results are in good agreements with those measured in simulations.

Comparisons between simulations results and observations obtained from experimental tests, shows that the method presented for modeling breakage, can help us to have a qualitative view about the effect of breakage phenomenon on behavior of granular materials.

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